

# p-Adic description of Higgs mechanism IV: elementary particle and hadron masses

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6.10. 1994

## Abstract

This paper is the fourth one in the series devoted to the calculation of particle masses in the framework of p-adic conformal field theory limit of Topological GeometroDynamics. In the third paper the masses of elementary fermions and bosons were calculated and in this paper these results are applied. The masses of charged leptons and  $W$  boson are predicted with accuracy better than one per cent. Weinberg angle is predicted to be  $\sin^2(\theta_W) = 3/8$  so that  $Z^0$  mass is 10 per cent too large. One can reproduce lepton and gauge boson masses correctly by taking into account Coulombic self energy associated with the interior of 3-surface and a small mixing of boundary topologies associated with charged leptons. The general mass formula for hadrons involves boundary contributions of quarks calculated in previous paper plus interior term consisting isospi-isopin, color magnetic spin-spin and color Coulombic terms. One must take also into account topological mixing of boundary topologies plus the mixing for primary condensate levels (quark spends part of time at lower condensate level). Topological mixing implies the nontriviality of CKM matrix. If (the moduli squared of) mixing matrix elements are rational numbers, mixing parameters must satisfy strong number theoretical constraints. It is possible to reproduce CKM matrix satisfying the empirical constraints but an open question is whether the number theoretic conditions can be satisfied. CP breaking is a number theoretical necessity. The mixing of  $u$  quark with  $c$  quark is large and solves the spin crisis of proton. The parameters associated with the interior  $O(p)$  contribution can be fixed by no Planck mass condition plus some empirical inputs.  $O(p)$  contribution to mass dominates for all hadrons except pion for which  $O(p^2)$  contribution from quark masses predicts mass correctly within few per cent.  $O(p^2)$  contribution determines isospin splitting: besides quark masses Coulomb interaction contributes to isospin splitting. In p-adic case also constant shifts coming from color magnetic spin-spin interaction, from isospin-isospin interaction, from the masses of sea partons,.. are important since modular mathematics is involved. Observed splittings can be reproduced but only few predictions (correct within experimental limits) are possible. The observed masses are reproduced with at most one per cent error. The only exception is top quark, whose mass is predicted to be about 5 times larger (3 times smaller) than the mass of observed top candidate

for  $k = 89$  ( $k = 97$ ). Arguments related to CP breaking seem to exclude  $k = 89$  and  $k = 97$  condensation levels. A rather plausible resolution of the problem is small mixing of  $k = 97$  and  $k = 89$  condensate levels implying increase in the mass of the top. An alternative possibility is that the observed top candidate in fact corresponds to lowest generation ( $g = 0$ ) quark of  $M_{89}$  physics obtained by scaling up the ordinary  $M_{107}$  physics by the ratio  $\sqrt{M_{107}/M_{89}} \simeq 512$  of Mersenne primes. The resulting prediction for the mass of top candidate is correct. The third alternative is that mixing model is correct and  $M_{89}$  mesons with nearly same energy as  $t\bar{t}$  mesons are also produced: this might explain the reported anomalies in the decay characteristics.

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# 1 Introduction

This is the fourth paper in series devoted to the p-adic description of Higgs mechanism in TGD. In the first paper the general formulation of p-adic conformal field theory limit of TGD was proposed and reader is suggested to read the introduction of this paper for general background. In paper III the calculations of elementary fermion and boson masses were carried out. In this paper the results of the calculation will be analyzed in detail and will be extended to a model predicting hadron masses with one per cent accuracy.

The predictions of TGD for lepton and gauge bosons masses are correct with error smaller than one percent except for  $Z^0$  boson, for which mass is too large by about 10 per cent. These discrepancies can be understood as following from the neglect of Coulombic self energy and small mixing of boundary topologies for leptons and when these effects are taken into account the masses of leptons and intermediate gauge bosons can be reproduced exactly. A good guess for renormalized Weinberg angle comes from the requirement that both  $\sin(\theta_W)$  and  $\cos(\theta_W)$  are rational numbers and therefore correspond to Pythagorean triangle. For minimal Pythagorean triangle one obtains the prediction  $\sin^2(\theta_W) = 8^2/17^2 \simeq 0.2215$  in accordance with the value deduced from  $\nu N$  scattering experiments [Arroyo *et al*].

TGD predicts also colored excitations of leptons and quarks. The existence of colored leptons is in accordance with the leptopion hypothesis [Pitkänen<sub>1</sub>, Pitkänen and Mähönen] used to explain the anomalous  $e^+e^-$  production observed in heavy ion collisions. On the other hand,  $Z^0$  decay widths plus the cherished hypothesis of asymptotic freedom seem to require these excitations to be very massive (to condense on condensate level with small  $p$ ). These issues will be considered in the fifth paper of the series.

Hadron mass formula involves several contributions.

- a) The contribution of valence and sea quarks. Thermodynamical equilibrium assumption together with quantization of chemical potentials at low temperature limit implies that sea partons give only  $O(p^2)$  contribution to masses. The masses of quarks were calculated in previous paper and mass formulas are almost identical with leptonic mass formulas.
- b) The interior  $O(p)$  contribution coming from color magnetic spin-spin interaction, electroweak isospin-isospin interaction plus color Coulombic interaction. The assumption of strong electroweak isospin-isospin interaction means

deviation from the picture of standard model. In order  $O(p)$  these contributions are parametrized by small integers and few empirical inputs plus no Planck mass condition fix the values of these integers completely.

c) Topological mixing of boundary topologies must be taken into account and few empirical inputs fix the mixing scenario essentially uniquely predicting Cabibbo angle correctly. The requirement that the moduli squared for U and D and CKM matrix elements are rational numbers implies strong number theoretical conditions on the parameters appearing in the mixing matrices for U and D type quarks and it is an open problem whether these conditions can be satisfied. An even stronger condition is the rationality of U,D and CKM matrices. CKM matrix satisfying empirical constraints can be found with essentially unique values of mixing parameters and a solution of spin crisis of proton follows as a byproduct. What happens is that  $u$  quark mixes unexpectedly strongly with  $c$  quark so that  $g = 0$  contribution to the spin of proton becomes of order  $1/3$ . CP breaking is a number theoretic necessity and comes out correctly as a prediction, when the values of various parameters are determined from the empirical estimates for the moduli squared of CKM matrix elements.

d) Also the mixing of primary condensate levels is needed in order to understand the lowering of the masses of hadrons  $b$  or  $c$  type quark plus  $u,d$  or  $s$  quark. The mixing of primary condensate level must be assumed: for instance,  $b$  quark (level  $k = 103$  with  $p \simeq 2^k$ ,  $k$  prime ) spends fraction of time as condensed on  $u,d$  or  $s$  quark (level  $k = 107$ ) and this lowers the effective  $b$  quark mass in hadrons containing  $u,d$  or  $s$  quarks.

e) The second order term in hadronic mass squared receives contributions from second order terms in quark masses, from the masses of sea partons, from electromagnetic and color magnetic spin-spin interaction, electroweak isospin-isospin interaction and color Coulombic and ordinary Coulombic interaction. Although quark masses and electromagnetic spin-spin interaction give the only contributions depending on isospin, the constant multiplet shifts implied by the other contributions are important in  $p$ -adic context since delicate modulo mathematics is involved. Since so many effects are involved, only a few predictions are possible at this stage.

f) In general the predictions are typically correct within one per cent. There is only single notable exception. Top quark condensed on condensate level  $k = 89$  ( $k = 97$ ) is predicted to have mass about 5 times larger (3 times smaller) than the mass of the observed top candidate. Arguments related

to CP breaking seem to force the top quark mass to be the mass of the observed top candidate and small condensate level mixing can indeed explain top mass.  $g = 0$  quarks of  $M_{89}$  hadron physics, which is just scaled up copy of  $M_{107}$  physics (predicted to exist in [Pitkänen<sub>b</sub>]) have however masses quite near to the mass of the top candidate and production of  $M_{89}$  hadrons might explain the reported anomalies in the production and decay of top candidate. The experimental signatures of new hadron physics will be discussed in the fifth paper of the series together with the problems related to the possible existence of light colored excitations of leptons and quarks.

## 2 Lepton masses

If interior contributions are neglected fermion mass squared is sum of cm and modular contributions related to the boundary component

$$M^2 = M^2(cm) + M^2(mod) \quad (1)$$

'cm' refers to the cm of boundary component. cm contribution is same for all families and depends on isospin of lepton. 'mod' refers to the contribution of modular degrees of freedom associated with boundary component. There is in principle present also a contribution coming from the interior of leptonic 3-surface and this contribution can be identified tentatively as electroweak self interaction energy: this contribution turns out to be smaller than percent as also suggested by dimensional considerations.

Modular contribution is given by the formula

$$\begin{aligned} M^2(mod, g) &= k(F)2N(g)g\frac{M_0^2}{p} \\ k(F) &= \frac{3}{2} \\ k(mod) &= 1 \end{aligned} \quad (2)$$

and has no dependence on electroweak and other standard quantum numbers.



## 2.1 Charged lepton masses

For charged leptons cm contribution is under rather general assumptions about prime  $p$  given by the formula

$$M^2(cm) = (5 + \frac{2}{3}) \frac{M_0^2}{p(L)} \quad (3)$$

The explicit form of the mass formula in case of charged leptons is

$$\begin{aligned} M^2(g, p) &= (5 + \frac{2}{3} + 3N(g)g) \frac{M_0^2}{p} \\ N(g) &= 2^{g-1}(2^g + 1) \end{aligned} \quad (4)$$

The formula gives following prediction for lepton masses

$$\begin{aligned} M^2(e) &= (5 + \frac{2}{3}) \frac{M_0^2}{M_{127}} \\ M^2(\mu) &= (14 + \frac{2}{3}) \frac{M_0^2}{p(\mu)}, \quad p(\mu) \simeq 2^{113} \\ M^2(\tau) &= (65 + \frac{2}{3}) \frac{M_0^2}{M_{107}} \end{aligned} \quad (5)$$

The relative errors of lepton masses are below one percent as the table shows.

particle	$m_{exp}/MeV$	$m_{pred}/MeV$	rel. error/%
e	0.5110034	0.5110034	0
$\mu$	105.659	105.229	-0.4
$\tau$	1776.9	1781.3	+0.2
$W$	80200	79582	-0.7
$Z$	91151	100664	10.0

Table 2.1. Predictions for masses of charged leptons and intermediate gauge bosons.

W/e mass ratio is predicted with smaller than one per cent error but  $Z^0$  mass is too large by ten per cent.

## 2.2 Effects of Coulombic self interaction and topology mixing on charged lepton masses

The predictions for the masses of  $\mu$  and  $\tau$  as well as  $W$  boson are too small by less than one per cent. The probable reasons for the discrepancy are the neglect of the interior contribution to mass squared and the mixing of boundary topologies.

a) The first guess is that interior contribution must corresponds to the electroweak self interaction energy of the particle, which is inversely proportional to p-adic length scale  $L(p) \frac{\sqrt{p}}{M_0}$ . The general form of the contribution to real mass is expected to be  $\Delta M(Coul) \sim \alpha_{em} \frac{M_0}{\sqrt{p(L)}}$ . This motivates the assumption that the general p-adic form of the Coulombic correction is

$$\Delta M^2(Coul) = k(Coul) \alpha_{em} p^2 \equiv K p^2 \quad (6)$$

where the parameter  $k(Coul)$  is same for all particles of same charge. For  $W/e$  and  $\tau/e$  mass ratio the sign of the effect is correct. This cannot be the whole explanation: the point is that the fraction of Coulombic energy from total energy is smaller for muon than for electron and therefore the muon/electron mass ratio tends to decrease rather than increase.

b) Second mechanism is related to the mixing of boundary topologies, which turns out to have important role in quark physics. The mixing of  $g = 0, 1, 2$  topologies can affect the masses in second order only so that in p-adic formalism one must have  $\sin^2(\phi_i) = n_i^2 p$  for mixing angles: this is possible since  $\sin(\phi) \propto \sqrt{p}$  is possible if 4-dimensional algebraic extension is used. The most general mixing is of form familiar from Kobayashi-Maskawa matrix

$$\begin{bmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 \exp(i\delta) & c_1 c_2 s_3 - s_2 c_3 \exp(i\delta) \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 \exp(i\delta) & c_1 s_2 s_3 - c_2 c_3 \exp(i\delta) \end{bmatrix}$$

The sines  $s_i$  of various angles are of form  $n_i \sqrt{p}$  and phase factor is assumed to be  $\epsilon = \pm 1$ . In extremely good approximation the changes in p-adic mass squared are given by

$$\Delta X(e) = (K + 9k_1) p^2$$

$$\begin{aligned}
\Delta X(\mu) &= (K - 9k_1 + 51k_2)p^2 \\
\Delta X(\tau) &= (K + 60k_2 - 9k_3)p^2 \\
k_1 &= n_1^2 \\
k_2 &= (n_2 - \epsilon n_3)^2 \\
k_3 &= (n_2 + \epsilon n_3)^2
\end{aligned}
\tag{7}$$

c) For electron the the contributions coming from Coulomb energy and mixing are compensated by the change in the mass scale  $m_0^2$ .

$$\begin{aligned}
(m_0^2)_R &\rightarrow \frac{5 + 23}{5 + \frac{2}{3} + \Delta(e)} \\
\Delta X(e) &= ((K + 9k_1)p^2)_R
\end{aligned}
\tag{8}$$

The latter formula holds true if  $K + k_1$  is finite sum of powers of two.

d) If one assumes that also the contributions to muon and tau masses come in powers of two one can write for the real counterparts of the masses the following expressions

$$\frac{m_2(L, exp)}{m^2(L, 0)} = \frac{s(L) + X(L) + \Delta(L)}{s(L) + X(L)} \frac{5 + \frac{2}{3}}{5 + \frac{2}{3} + \Delta X(e)}
\tag{9}$$

where  $m^2(L, 0)$  is the prediction for lepton mass without any corrections.

e) Additional information to fix the values of the parameters is needed and one one might think that the requirement that W/e mass ratio is predicted correctly gives one constraint but this is not all that is needed. The constraint comes from the following observation. It is natural to assume that hadronic isospin splittings are of second order in  $p$  so that  $s(H)$  is same for all hadrons inside isospin multiplets. If one fits the hadron masses using general formula provided by Super Virasoro representations one finds that this requirement is satisfied if the condition

$$\Delta X(e) \geq 0.0155 \quad (10)$$

is satisfied. The nearest power of two is

$$\Delta X(e) = 2^{-6} \simeq 0.0156 \quad (11)$$

and leads to an excellent prediction for  $Z$  boson mass

$$m(Z, pred) \rightarrow \frac{5 + \frac{2}{3}}{5 + \frac{2}{3} + \Delta(e)} m(Z, 0) \simeq .9986 m(Z, exp) \quad (12)$$

with error of 0.24 per cent and in the absence of other corrections to  $Z^0$  mass this means that the identification of mass scale must be correct.

f)  $W$  mass is very sensitive to the exact value of the Coulombic correction  $K$  and the requirement that  $W$  mass is correct fixes the value of  $K$

$$\begin{aligned} m(W) &= \sqrt{1 + 2K} \frac{5 + \frac{2}{3}}{5 + \frac{2}{3} + \Delta X(e)} m(W, 0) \\ K &= 2^{-4}(1 + 2^{-2} + 2^{-5} + 2^{-7} + 2^{-8}) \simeq 0.0808 \end{aligned} \quad (13)$$

g) Using the value  $\Delta X(e)$  and  $K$  one can deduce the values of the mixing parameters  $k_1, k_2$  and  $k_3$  from leptonic masses and one obtains

$$\begin{aligned} 9k_1 &= \Delta X(e) - K = -(2^{-4}(1 + 2^{-5} + 2^{-7} + 2^{-8})) \simeq -0.0652 \\ 51k_2 &= \Delta X(e) - 2K + \Delta X(\mu) = 2^{-7}(1 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-5}) \simeq .0149 \\ 9k_3 &= \Delta X(\tau) - \frac{60\Delta X(\mu)}{51} - \frac{60\Delta X(e)}{51} + (\frac{120\Delta X(e)}{51} - 1)K \\ &= -2^{-2}(1 + 2^{-1} + 2^{-3} + 2^{-6} + 2^{-7} + 2^{-8} + 2^{-9} + 2^{-11}) \simeq -0.4293 \end{aligned} \quad (14)$$

The negative values of  $k_1 = n_1^2$  and  $k_3$  do not mean any problem since  $k_1$  is equivalent with  $p - k_1$ , which is positive quantity. The real counterpart of  $s_1^2$  can be defined as  $(s_1^2)_R / ((s_1^2)_R + (c_1^2)_R)$  and corresponds to a rather large angle.

## 2.3 Neutrino masses

The estimation of neutrino masses is difficult at this stage since the prediction of the primary condensation level is not yet possible. The cosmological bounds for neutrino masses however help to put upper bounds on the masses. If one takes seriously the preliminary data on neutrino mass measurement [Louis] and the explanation of the atmospheric  $\nu$ -deficit in terms of  $\nu_\mu - \nu_\tau$  mixing [Fukuda *et al*, Borodovsky *et al*] one can deduce that the condensation level of all neutrinos is  $k = 163$  and deduce information about the CKM matrix associated with neutrinos so that mass predictions become unique.

For neutrinos the expression for the cm contribution is given by the formula

$$\begin{aligned} M^2(cm) &= s \frac{M_0^2}{p} + X \\ s &= 3 \\ X &= (-\frac{1}{2}p^2)_R \end{aligned} \tag{15}$$

where  $M_0^2$  is universal mass scale. Second order contribution depends on the value of  $p$  since the p-adic inverse of the number 20 is contained in the formula (sub-index  $R$  denotes real counterpart of p-adic number). Under rather general assumptions about  $p(\nu)$  the contribution can be written as

$$M^2(int, \nu) = (3 + \frac{1}{2}) \frac{M_0^2}{p(\nu)} \tag{16}$$

With the above described assumptions one has the following mass formula for neutrinos

$$\begin{aligned} M^2(\nu) &= A(\nu) \left( \frac{M_0^2}{p(\nu)} \right) \\ A(\nu_e) &= 3 + \frac{1}{2} \\ A(\nu_\mu) &= 12 + \frac{1}{2} \\ A(\nu_\tau) &= 63 + \frac{1}{2} \end{aligned} \tag{17}$$

In [Pitkänen<sub>b</sub>] it was suggested that the primary condensation levels of neutrinos correspond to primes near prime powers of 2:  $p \simeq 2^k$ ,  $k$  prime. Together with known bounds for neutrino masses this leaves only few possibilities for the values of neutrino masses.

Arguments based on 2-adic description of Higgs mechanism (,which do not hold true in present scenario) suggest that the most probable condensation levels correspond to the primes

$$\begin{aligned} k(\nu_e) &= 163 \\ k(\nu_\mu) &= 149 \\ k(\nu_\tau) &= 137 \end{aligned} \tag{18}$$

With this assumption

$$\begin{aligned} m(\nu_e, 163) &\simeq 1.485 \text{ eV} \\ m(\nu_\mu, 149) &\simeq 0.305 \text{ keV} \\ m(\nu_\tau, 137) &\simeq 0.053 \text{ MeV} \end{aligned} \tag{19}$$

These estimates are consistent with the recent upper bounds [Dolgov and Rothstein] of order 10 eV, 270 keV and 0.3 MeV for  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  respectively.

The recent measurement [Louis] suggests that the masses of both electron and muon neutrinos are in the range .5 – 5 eV and that mass squared difference is  $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$  is between .25 – 25 eV<sup>2</sup>. This requires that  $\nu_\mu$  and  $\nu_e$  have common condensation level  $k = 163$  (in analogy with d and s quarks). The resulting prediction (in absence of mixing) for  $\nu_\mu$  mass is  $m(\nu_\mu) = 2.38 \text{ eV}$  and  $\Delta m^2 \sim 3 \text{ eV}^2$ . The interpretation of the experiment is based on small CKM mixing of muon and electron neutrinos having nearly degenerate masses.

The second source of information is the atmospheric  $\nu_\mu/\nu_e$  ratio, which is roughly by a factor 2 smaller than predicted by standard model [Fukuda *et al*]. A possible explanation is the CKM mixing of muon neutrino with  $\tau$ -neutrino, where as the mixing with electron neutrino is excluded as an explanation. The latest results from Kamiokande [Fukuda *et al*] are in accordance with the mixing  $m^2(\nu_\tau) - m^2(\nu_\mu) \simeq 1.6 \cdot 10^{-2} \text{ eV}^2$  and mixing angle  $\sin^2(2\theta) = 1.0$ :

also the zenith angle dependence of the ratio is in accordance with the mixing interpretation. If this result is taken seriously then the only conclusion is that all three neutrinos condense on  $k = 163$  level with  $L(163) \simeq 7.6E-7 m$ . Combining the result of [Louis] with the upper bound  $\sin^2(2\phi) < .003$  for  $e - \mu$  mixing angle at large  $\Delta m^2$  limit [Borodovsky *et al*] one can conclude that  $e - \mu$  mixing is small as compared with  $\mu - \tau$  mixing.

From the general form of the mass formula at  $k = 163$  level and from the formula for the mixed masses

$$\begin{aligned} M(\nu_\mu)_{mix} &= \cos^2(\theta)M^2(\nu_\mu) + \sin^2(\theta)M^2(\nu_\tau) \\ M^2(\nu_\tau)_{mix} &= \sin^2(\theta)M^2(\nu_\mu) + \cos^2(\theta)M^2(\nu_\tau) \end{aligned} \quad (20)$$

one can deduce the value of mixing angle  $\theta$  assuming  $\delta M^2(\tau - \mu) = 1.6E - 2 \text{ eV}^2$  to be given. The predictions are

$$\begin{aligned} m(\nu_e) &= 1.485 \text{ eV} \\ m(\nu_\mu)_{mix} &= 4.895 \text{ eV} \\ m(\nu_\tau)_{mix} &= 4.891 \text{ eV} \\ \sin^2(2\theta) &= .99999949 \\ \Delta m^2(\mu - e) &= 21.7 \text{ eV}^2 \end{aligned} \quad (21)$$

The prediction of the mixing angle is in agreement with the experimental results [Fukuda *et al*]. The prediction of mass for muon type neutrino is slightly below the upper bound  $5 \text{ eV}$  given by the previously mentioned experiment [Louis]. Also the prediction for  $\Delta m^2(e - \mu)$  is within preliminary experimental bounds [Louis]. A more general neutrino mixing matrix  $D$  of form

	e	$\mu$	$\tau$
$\nu_e$	$c_1$	$s_1$	0
$\nu_\mu$	$-s_1c_2$	$c_1c_2$	$c_2$
$\nu_\tau$	$-s_1c_2$	$c_1c_2$	$-c_2$

where  $c_2 = 1/\sqrt{2}$  (masses of  $\mu$  and  $\tau$  neutrinos are assumed to be identical) makes  $\nu_e - \nu_\mu$  mass splitting smaller. For  $s_1^2 = 1$  one has  $\Delta m^2(e - \mu) =$

$2.5 \cdot 6 \text{ eV}^2$ . Large value of  $s_1$  is questionable since it is not at all obvious whether charged lepton mixings can be such that  $\nu_\mu - e$  element of CKM matrix is of order  $10^{-3}$  as required by [Borodovsky *et al*].

## 2.4 Color excited leptons

In the previous paper it was found that TGD predicts also color excited leptons and quarks. Both charged and neutrinos allow massless 10 and  $\bar{10}$  color multiplets and neutrinos also 27 dimensional color multiplet. Also  $U$  type quarks allow excitations created by operators belonging to 10 and  $\bar{10}$  color multiplets. In previous papers the existence of color octet (rather than decuplet) excitations of leptons were used to explain the anomalous  $e^+e^-$  pairs observed in the heavy ion collisions [Pitkänen<sub>1</sub>, Pitkänen and Mähoenen]: the resonances were identified as color bound states of colored leptons. Effectively the hypothesis means the existence of a new branch of Physics at the energy scale of one MeV. The decay widths of the intermediate gauge bosons seem to exclude light exotic fermions and in the fourth paper of the series it will be shown how p-adic unitarity allows to avoid this restriction by replacing the condition on the number of light fermions with modulo type condition.

## 3 Masses of elementary bosons

The explicit calculations show that the masses of gauge bosons are in qualitative accordance with expectations assuming  $T(ew) = 1/2$  for electroweak gauge bosons and  $T = 1$  for exotic bosons.

- a) Gluons are predicted to be exactly massless to order  $O(p^2)$  whereas photon has extremely small thermal mass. The requirement that photon is massless fixes the parameter  $k(B)$  to  $k(B) = 3/2$ , which is the ratio of Ramond and ground state N-S mass scales. The difference between N-S and Ramond string tensions implies that ground state mass scales are identical for these representations.
- b) Weinberg angle is predicted to be  $\sin^2(\theta_W) = 3/8$ , typical value for the parameter in GUTs at symmetry limit. Secondary topological condensations are expected to renormalize the value of the Weinberg angle in TGD.
- c) The prediction for the ratio of  $W$  boson and electron masses is too small by  $-0.7$  per cent. Main discrepancy is related to the too high value of  $Z^0$  mass



implied by the too large value of Weinberg angle. It was already found that these discrepancies are disappear, when topological mixing effects for leptons and Coulomb energy are taken into account. The requirement that  $\sin(\theta_W)$  and  $\cos(\theta_W)$  are rational numbers is very attractive and implies that Weinberg angle corresponds to Pythagorean triangle. Requiring that the sides of this triangle are as small as possible one obtains the triangle  $(r, s) = (4, 1)$  and  $P = 8^2/17^2 \simeq 0.2215$ . For this value of Weinberg angle  $Z^0$  mass is predicted within one per cent error correctly. The Pythagorean value of Weinberg angle is within experimental uncertainties identical with the value of Weinberg angle  $P = 0.2218 \pm 0.0059$  determined from neutrino-nucleon scattering [Arroyo *et al*] and slightly smaller than the value  $0.2255 \pm 0.0019$  determined from LEP precision experiments [Schaile].

boson	$M_{obs}/MeV$	$M_{pred}/MeV$	error/%
$\gamma$	0	0	0
gluon	0	0	0
$W$	80200	79582	-0.8
$Z$	91151	100664	10

Table 2.2. Masses of nonexotic gauge bosons without corrections coming from topological mixing of leptons and Coulomb energy.

d) There is large number of candidates for exotic bosons. If one assumes  $T = 1$  for exotic bosons most states possess either Planck mass or are massive but 'light'. What is remarkable is the absence of massless noncolored exotic bosons for  $T = 1$ .

spin	charge operator	$M^2(T = 1)$	$M^2(T = 1/2)$
0	$I_{L/R}^3$	Planck mass	$\frac{1}{2p}$
0	$I^\pm$	Planck mass	$\frac{1}{2p}$
0	$I_{L/R}^3 Q_K$	$\frac{3}{p}$	0
0	$I^\pm Q_K$	Planck mass	$\frac{1}{2p}$
1	1	Planck mass	$\frac{1}{2p}$

Table 2.3. Masses and couplings of noncolored light exotic bosons for  $T = 1$  and  $T = 1/2$ . Charge operator tells how the boson in question

couples to matter. For  $T = 1/2$  the states with charge operator  $I_{L/R}^3 Q_K$  are essentially massless for large values of  $p$  and some additional light states become possible.

spin	charge operator	$D$	$M^2(T = 1)$	$M^2(T = 1/2)$
0	$I^\pm Q_K$	8	$\frac{2}{p}$	0
1	$I^\pm$	8	Planck mass	$\frac{1}{2p}$
1	$I_{L/R}^3 Q_K$	8	Planck mass	$\frac{1}{2p}$
1	$I^\pm Q_K$	8	Planck mass	$(\frac{3p}{10})_{R\frac{1}{p}}$
0	$I^\pm Q_K$	10, 10	$\frac{3}{p}$	0
0	1	10, 10	0	0
1	$I^\pm$	10, 10	$\frac{3}{p}$	0
1	$I_{R/L}^3 Q_K$	10, 10	$\frac{3}{p}$	0
0	$I^\pm$	27	Planck mass	$\frac{1}{2p}$
0	$I_{R/L}^3 Q_K$	27	Planck mass	$\frac{1}{2p}$
1	$I_{R/L}^3$	27	Planck mass	$\frac{1}{2p}$
1	$Q_K$	27	Planck mass	$\frac{1}{2p}$
0	$I_{R/L}^3$	27	0	0
0	$Q_K$	27	0	0
1	1	27	0	0

Table 2.4. Masses and couplings of colored exotic bosons for  $T = 1$  and  $T = 1/2$ . The last two massless bosons are doubly degenerate due to occurrence of two 27-plets with conformal weight  $n = 2$ .  $T = 1/2$  is physically possible alternative since no long range forces are implied.

e) There is distinct possibility for the higher generation gauge bosons. The higher generations of intermediate gauge bosons should have condensation level with  $p < M_{89}$ : the first candidate is  $p \simeq 2^{83}$ : the masses of  $g = 1$  bosons are for  $k = 83$   $m(W(g = 1)) \simeq 24.0m(W)$  and  $m(Z(g = 1)) \simeq 19.6m(Z)$ . The next possibilities are  $k = 79, 73, 71$  and  $M_{61}$ . Also photon could allow higher generation excitations: they would be certainly massive and have mass  $\sqrt{2N(g)g/p(\gamma)}$ .

f) There is no boson identifiable as Higgs doublet: doublet is excluded already by the fact that  $N - S$  representations do not allow isospin doublets. The

result is in accordance with earlier suggestions. The TGD:eish counterpart of Higgs field is Virasoro generator  $L_0$ , which develops vacuum expectation value in conformal symmetry breaking description of Higgs mechanism.

d) Graviton is not possible in the proposed scenario. The largest possible values of spin and isospin are  $J = 1$  and  $I = 1$ . This result is in accordance with the basic assumptions about p-adic TGD as flat spacetime limit of Quantum TGD. Of course, one could try to construct the counterpart of closed string model (closed string is replaced by two sheets of flat spacetime glued along boundaries) to get graviton but there is no guarantee that spin 2 massless state having  $D(1) = 0$  is obtained.

## 4 Hadron masses

The general mass formula can be written as sum of interior and boundary contributions:

$$\begin{aligned} M^2(h) &= M^2(\delta) + M^2(int) \\ M^2(\delta) &= M^2(cm) + M^2(mod) \end{aligned} \tag{22}$$

The contribution  $M^2(mod)$  from the modular degrees of freedom is same as for leptons and sum over the contributions of hadronic valence quarks. Boundary cm contribution can be evaluated using p-adic thermodynamics and it turns out that the contribution is in first order just the sum of single quark contributions and of same form as for leptons. In second order mass squared is not any more additive. For D quark second order contribution is same as for charged leptons but for U quark the contribution is not identical with that of neutrino.

The mere boundary contribution gives mass formula, which is qualitatively correct but it is obvious that something is missing. There are several mixing effects present.

a) It turns out that modular contribution dominates for heavier quarks and predicts too large masses for hadrons unless mixing of boundary topologies  $U$  and  $D$  type quarks is allowed. Different topological mixing for  $U$  and  $D$  type quarks implies in turn nontriviality of Kobayashi-Maskawa matrix. Although one cannot predict at this stage the mixing angles one can use

some empirical input together with number theoretical consideration to derive strong constraints on the values for mixing angles. First, the change  $\Delta s$  of quark is small integer. p-Adic unitarity plus the assumption that  $|U_{ij}|^2$  are rational numbers (allowing interpretation either as real or p-adic numbers) implies that the mixing matrix is determined by two small parameters. More stringent condition is rationality of  $U_{ij}$ . Cabibbo angle is predicted correctly within experimental uncertainties from the requirement that  $u$  and  $d$  quarks have identical masses in order  $O(p)$  and the orders of magnitude for the small elements of the KM matrix are predicted correctly. Even the necessity of nontrivial phase factors leading to CP breaking is forced by number theory. It should be noticed that in TGD framework the mixings of  $U$  and  $D$  type quarks are both observable, not only the difference between these mixings as in standard model.

b) Mixing of boundary topologies is not enough: the masses of hadrons containing one charmed or bottom quark are systematically too heavy whereas masses of diagonal mesons of type  $c\bar{c}$  and  $b\bar{b}$  are predicted quite satisfactorily. The explanation is mixing of primary condensation level for  $c$  and  $b$  quark.  $c$  and  $b$  do not spend all their time at  $k = 103$  condensation level but condenses now and then on  $u, d, s$  quark having  $k = 107$ . Condensation is not possible in diagonal mesons since no  $u, d$  or  $s$  type quark is present.

c) Third type of mixing effect is mixing of neutral pseudoscalar mesons made possible by annihilation to two-gluon intermediate state. The mixing is important for mesons such as  $\eta, \eta\eta_c$  and in the absence of the mixing  $\eta$  and  $\pi$  would have identical masses.

The remaining hitherto identified  $O(p)$  contributions come from the interior of hadron.

a) Color Coulombic force is strong and is expected to give sizable contributions to mass squared. Since  $T = 1$  turns out to be the only sensible choice for the p-adic temperature the value of  $s(M)$  should be nonvanishing for all mesons. For pion the mass fit however gives  $s(\pi) = 0$ . This means that interior contribution to mass squared of mesons must be of the order  $O(p)$ , be negative and integer value and cancel the contribution coming from quark masses in pion. Contribution is also same for all baryons/mesons.

b) Color magnetic hyper fine splitting gives nice explanation for  $\rho - \pi$ ,  $K^* - K$ ,  $N - \Delta$  mass splittings and must be included also now in order  $O(p)$ . Number theoretic considerations force this contribution to be sum of integer valued

contributions. The integers associated with each quark pair are in principle different (contribution is inversely proportional to the product of quark masses in first order QCD).

c) It turns out necessary to assume isospin-isospin interaction between baryonic quarks of same generation in order to understand masses of doubly strange baryons. The general form of the splitting is same as for color magnetic interaction. For mesons this interaction is in principle also present but turns out to vanish in first order. Also isospin isospin interaction between different generations is in principle present but can be assumed to vanish.

Second order contributions to hadron masses are small as compared to  $O(p)$  contributions with pion forming exception: the prediction of pion mass gives a crucial test for the model in second order. Perhaps the most important second order contribution comes from boundary cm degrees of freedom and this contribution is isospin dependent and depends on the electroweak isospin of the state only. This contribution is identical for neutral and charged pion and about 3 per cent smaller than pion mass. The inspection of hadronic isospin splittings shows that this contribution cannot be the only one. The ordinary electromagnetic Coulomb interaction is expected to give an additional negative second order contributions.

The contribution of sea partons is expected to be present, too. The case of pion suggests that contribution is small as compared to second order contribution given by quark masses. An attractive possibility is that thermodynamic description applies to sea partons, too. In thermodynamic description one must introduce chemical potential for each parton type and in low temperature limit  $\mu/T$  is quantized to integers by p-adic existence requirement for Boltzmann weight:  $\mu/T = n$ . This fixes entirely the number distribution for sea apart from the parton distribution functions for longitudinal momentum fraction (recall that quarks are most of the time massless particles!). For nonvanishing value of chemical potential sea partons give only  $O(p^2)$  contribution to mass squared.

It must be admitted that TGD doesn't yet predict the various parameters needed for understanding hadronic masses. It must be emphasized however that by number theory all these parameters are just integers with very limited range of allowed values and simple physical constraints turn out to make the choice between the alternatives unique. The errors of resulting simple mass

formula are about one or two per cent.

#### 4.1 Evaluation of boundary cm contribution to hadron mass

The contribution coming from the cm of boundary component can be evaluated using p-adic thermodynamics for the tensor product of quark Super Virasoro representations. Since quarks correspond to different boundary components and Super Virasoro generators correspond to infinitesimal conformal transformations acting in each boundary component separately it is obvious that quarks must obey Super Virasoro conditions separately. Since the value of first order contribution to mass squared is typically of the order of Planck mass it would be highly desirable if quarks would obey same mass formulas as leptons in first order. The necessary condition for this is that isospin splitting of quark mass squared is described as the shift of vacuum weight  $h$  using same formula for leptons and quarks

$$\begin{aligned} h(U) &= -2 - 2/9 + \frac{Q_K^2}{2} = -2 \\ h(D) &= -1 \end{aligned} \tag{23}$$

so that essentially same operators create  $M^2 = 3/2$  states for  $U$  and  $\nu$  and  $D$  and  $e$  respectively.

The general p-adic mass formula was derived from p-adic thermodynamics in previous paper and is given by

$$\begin{aligned} \frac{M^2(h)}{M_0^2} &= sp + Xp^2 \\ s &= k(F) \frac{D(h, 1)}{D(h, 0)} \\ X &= k(F) (2D(h, 2) - (\frac{D(h, 1)}{D(h, 0)})^2) \\ k(F) &= 3/2 \end{aligned} \tag{24}$$

Here  $D(h, n)$  denotes the degeneracy of state with mass  $M^2 = 3/2n$  created by gauge invariant operators of conformal weight  $n$  from vacuum.

For hadron state with mass  $M^2 = 0$  has degeneracy given by the product of single quark degeneracies  $D(h, 0)$

$$D(h, 0) = \prod D(q_i, 0) \quad (25)$$

$M^2 = 3/2$  states of hadron are obtained by exciting one of the quarks to this state while other quarks remain massless. This gives

$$s(h) = \sum_i \frac{D(1, i)}{D(0, i)} = \sum_i s(q_i) \quad (26)$$

so that mass squared is additive in first order. This result implies that hadrons are light. The allowance of color excitations for individual quarks would change the degeneracies of both  $M^2 = 0$  and  $M^2 = 1$  states and as a result most hadrons would have Planck mass.

The values of the parameters  $s(q)$

$$\begin{aligned} s(U) &= s(\nu) = 3 \\ s(D) &= s(e) = 5 \end{aligned} \quad (27)$$

were derived in previous paper and are same as for leptons. The first formula implies large isospin splitting for  $u$  and  $d$  quark and the mixing of boundary topologies for  $U$  and  $D$  quarks must be such that effective values of  $s(U)$  and  $s(D)$  are identical. The requirement implies essentially the experimental value for Cabibbo angle.

$M^3 = 3$  excitations correspond to states where single quark is excited to  $M^2 = 3$  state or to states where two quarks are excited to  $M^2 = 3/2$  state both. This means that mass squared is not additive in second order. The general expression for  $D(h, 2)$  is given by

$$D(h, 2) = \sum_i D(2, q_i) + \sum_{pairs (i,j)} D(q_i, 1)D(q_j, 1) \quad (28)$$

and one obtains for the coefficient of the second order contribution the following expression

$$\begin{aligned}
X(h) &= \sum_i X(q_i) + \Delta(h) \\
X(q_i) &= \frac{D(q_i, 2)}{D(q_i, 0)} - \sum_i \left( \frac{D(q_i, 1)}{D(q_i, 0)} \right)^2 \\
\Delta(h) &= \sum_{pairs} \sum \frac{D(q_i, 2)}{D(q_i, 0)} + \sum_{pairs} \frac{D(q_i, 1)D(q_j, 1)}{D(q_i, 0)D(q_j, 0)} - \left( \sum \frac{D(q_i, 1)}{D(q_i, 0)} \right)^2 + \sum_i \left( \frac{D(q_i, 1)}{D(q_i, 0)} \right)^2
\end{aligned} \tag{29}$$

The term  $\Delta$  gives deviation from the simple additive formula for mass squared. The values of the coefficients  $D(U, n)$  and  $D(D, n)$  were derived in previous paper and are given by the table below.

quark type	$D(0)$	$D(1)$	$D(2)$
U	40	80	8
D	12	40	80

Table 4.1. Degeneracies for various mass squared values for quarks.

As far as cm contribution to mass is considered there are only 4 nonequivalent combinations for baryonic quarks, namely  $UUU, UUD, UDD$  and  $DDD$ . For mesons there are 3 combinations  $U\bar{U}$ ,  $D\bar{D}$  and  $U\bar{D}$ . The integer part of  $X$  gives completely negligible contribution to mass squared in the absence of mixing of different quark configurations ( $D\bar{D}$  and  $U\bar{U}$  for neutral pion). The following table summarizes the values of first and second order terms  $X$  for these configurations. Also are listed the real counterparts for  $Xp^2$  obtained by using the canonical correspondence between p-adic and real numbers ( $\sum x_k p^k \rightarrow \sum x_k p^{-k}$ ).



hadron type	s	X	$p(Xp^2)_R$
UUU	9	$\frac{24}{30}$	$\frac{24}{60}$
UUD	12	$\frac{13}{30}$	$\frac{13}{60}$
UDD	13	$\frac{18}{30}$	$\frac{8}{60}$
DDD	15	0	0
$UU$	6	$\frac{6}{30}$	$\frac{4}{5}$
$DD$	10	$-\frac{10}{30}$	$\frac{1}{3}$
$UD$	8	$\frac{13}{30}$	$\frac{2}{15}$

Table 4.2. The values of the second order contribution to mass squared coming from quark masses for various quark combinatios.

The mass fit for pion shows that  $s(eff)$  must vanish for pion (due to interior contributions to mass squared) so that second order contribution should give entire pion mass. Neutral pion is superposition of form  $\sqrt{1/2}(U\bar{U} + D\bar{D})$  one obtains for the masses of neutral and charged pions the following expressions

$$\begin{aligned}
X(\pi^0) &= \frac{1}{2}(X(U\bar{U}) + X(D\bar{D})) = \left(\frac{3}{5} - \frac{1}{6}\right) = \frac{13}{30} \\
X(\pi^+) &= X(U\bar{D}) = \frac{13}{30}
\end{aligned} \tag{30}$$

It should be noticed that in the calculation of pion mass one cannot use directly the sum of parameters  $X$  associated with  $U\bar{U}$  and  $\bar{D}D$  since this would drop the possible half odd integer contribution to  $X_{eff}$ . It is remarkable that the predicted values are identical for neutral and charged pion. In order to calculate the real mass one must find the real counterpart of  $Xp^2 = (13p^2)/30$  by canonical correspondence between p-adic and real numbers. Using the formulas from the last appendix of previous paper one has  $Xp^2 \rightarrow X_R = 22/60p$  for  $p = M_{107}$ . The result of mass fit is  $X_R = 24/p$  for charged and  $X_R = 23/p$  for neutral pion. Errors are of order 3 per cent!

## 4.2 Contribution of modular degrees of freedom and primary condensation levels

Modular degrees of freedom give readily calculable real contribution  $M^2(mod) = 3gN(g)M_0^2/p(q)$  to the mass.  $p(q)$  is the prime associated with the primary condensation level of the quark and depends on quark. Since tensor product of Super Virasoro representations associated with boundary degrees of freedom is involved in calculation the boundary contributions of quarks to total mass squared are summed together already at p-adic level. The only possibility is that  $k = 107$  is common secondary condensation level of  $M_{107}$  hadron physics whereas primary condensation levels must be differ from  $k = 107$  for heavier quarks. The only possible primary condensation levels for quarks turn out to be following ( $p(q) \simeq 2^k$ ):

$$\begin{aligned} k(u) = k(d) = k(s) &= 107 \\ k(c) = k(b) &= 103 \\ k(t) &= 89 \text{ or } 97 \end{aligned} \tag{31}$$

The secondary and primary condensation levels of  $u, d, s$  quarks are identical. This sounds peculiar at first but is in accordance with the mass minimization mechanism suggested in second paper of series: if primary and secondary condensation levels have nearly identical  $p \simeq 2^k$  then second order contribution to mass can suffer large decrease in secondary condensation. In [Pitkänen<sub>b</sub>] the existence of an entire hierarchy of hadronic physics was suggested in the sense that there is hadronic physics associated with each Mersenne prime. In particular, quarks of each generation have replicas with scaled up masses.

## 4.3 Mixing of boundary topologies

In TGD the different mixings of boundaries topologies for  $U$  and  $D$  type quarks provide the fundamental mechanism for Cabibbo mixing and also CP breaking. In the determination of Kobayashi-Maskawa matrix one can use following conditions.

a) Mass squared expectation values in order  $O(p)$  for mixed states must be integers and the study of hadron mass spectrum leads to very stringent conditions on the values of these integers. Physical values for these integers imply

essentially correct value for Cabibbo angle provided  $U$  and  $D$  matrices differ only slightly from mixing matrices mixing only the two lowest generations.

b) The matrices  $U$  and  $D$  describing the mixing of  $U$  and  $D$  type boundary topologies are unitary in p-adic sense. The requirement that the moduli squared of the matrix elements are rational numbers is very attractive since it suggests equivalence of p-adic and real probability concepts and therefore could solve the conceptual problems related to the transition from p-adic to real regime. It must be however immediately added that rationality assumption for the probabilities defined by S-matrix turns out to be unphysical. Unitarity requirement is nontrivial since mass conditions give constraints for the squares of cosines and sines for various angle parameters, only. The requirement that sines and cosines parametrizing  $U$  and  $D$  matrices exist as p-adically real numbers is very strong and leads to the conclusion that  $U$  and  $D$  matrices contain only two small parameters.

c) One can require that the probabilities defined CKM matrix elements are also rational numbers or even that  $U, D$  and CKM matrices are rational and this gives very strong number theoretic conditions. The general solution for CKM rationality conditions can be found. Two of the angles appearing in  $U$  and  $D$  matrix correspond to Pythagorean triangles and the remaining two angles to triangles with integer valued shorter sides with sine and cosine allowing common irreducible rational phase. If the elements of  $U, D$  and CKM matrices are required to be rational all angle parameters correspond to Pythagorean triangles. This alternative is not excluded by number theoretic condition in the scenario providing a solution for the spin crisis of proton.

d) The requirement that Cabibbo angle has correct value fixes essentially uniquely the values of mixed mass squared for various mesonic quark generations. In real regime it is quite easy to reproduce KM-matrices satisfying experimental constraints by choosing appropriate values for the small parameters, one of which is CP breaking angle but the problem whether there actually exists any  $U$  and  $D$  matrices satisfying number theoretical conditions remains open at this stage.

#### 4.3.1 Do mesonic and baryonic quarks mix identically?

The attractive assumption that the mixing matrices for baryons and mesons are identical gives for Cabibbo angle the rough estimate  $\sin(\theta_c) \simeq 0.2236$ , which is slightly below the experimentally allowed range  $\sin(\theta_c) = 0.226 \pm$

0.002: mixing with third generation can slightly increase the value. This potential discrepancy could serve as motivation for asking whether the mixing matrices could be different for baryons and mesons (to be honest, the actual motivation was calculational error, which yielded quite too small value of Cabibbo angle!)

One could indeed seriously consider the possibility that topology mixing and perhaps even CKM matrices are different in baryons and mesons. The point is that it is the electroweak current, which contains the Cabibbo mixed quarks. The matrix elements of the electroweak and color currents are not changed between baryon states if one performs any unitary change of basis. In GUTS the mixing of  $U$  and  $D$  type quarks corresponds to global gauge transformation and therefore unitary transformation. In TGD the mixing of quarks is not a gauge symmetry since mass squared changes for quark but for all amplitudes expressible as matrix elements of currents there are no observable effects. The different mixings for mesons and baryons are not seen at the level of amplitudes by baryon number conservation at single particle level, say in semileptonic decays of baryons.

The first instance, where effects might be seen is  $B\bar{B}$  annihilation to meson pair but if the amplitude for this process can be described in terms of electroweak and colored currents there are no observable effects. This is not obvious in TGD. If dual diagrams provide more than a phenomenological description of nonperturbative aspects of this process then  $d$  and  $s$  quark lines running from meson to baryon must contain a vertex, where mixing angle and quark mass changes and empirical effects are predicted. Of course, the troublesome fact that mesonic and baryonic quark masses tend to be different also in naive quark model fit of hadron masses can be regarded as direct indication of correctness of the TGD:ish prediction. The appearance of mesonic Cabibbo mixing at the level of currents follows from the fact electroweak currents have same electroweak quantum number structure as vector mesons. For example, the electroweak couplings of the charged electroweak current to baryons at low energies can be in good approximation expressed using generalized vector dominance model: the emission of gauge boson from baryon decomposes to the emission of vector meson, which couples to gauge boson. This means that matrix elements of currents are proportional to the matrix elements of meson fields.

The calculations show that effective mass squared for strange quark is different for mesons and baryons in the optimal scenario so that baryonic and mesonic mixing matrices are different. CKM matrices are however essentially identical for the physical values of parameters.

#### 4.3.2 Topology mixing and quark masses

The requirement that hadronic mass spectrum is physical requires mixing of  $U$  and  $D$  type boundary topologies. The following arguments fix the effective values of the modular contribution  $s(\delta, q_i)$  to the mass squared.

- a) The smallness of isospin splitting for hadrons containing only  $u$  and  $d$  quarks implies that the effective value of the modular contribution  $s(\delta)$  must be same for  $u$  and  $d$  quark:  $s_{eff}(u) = s_{eff}(d)$  for all hadrons.
- b) The requirement that  $\Delta - p$  mass difference resulting from hyper fine splitting and isospin-isospin interaction is of correct size requires  $s_{eff}(u) = s_{eff}(d) = s = 8$  in baryons. If color binding energy is required to be non-vanishing for baryons as it is in mesons then  $s(d) = s(u) = 9$  is the only possibility.
- c) Proton- $\Lambda$  mass difference is not affected by colormagnetic spin-spin splitting nor isospi-isospin interaction and suggests strongly  $s_{eff}(s) = 16$  in baryons. Also  $\Omega$  mass suggests same. For mesons  $s_{eff}(s) \geq 13$  is forced by the need to obtain mixing between third and lower generations at all.  $K - \pi$  mass difference suggests  $s_{eff}(s) = 14$ . This raises the possibility that baryonic and mesonic mixing matrices are different: it turns out that for physical parameter values CKM matrix depends only very weakly on  $s_{eff}(s)$ .
- d) The masses  $c\bar{c}$  mesons come out correctly if one has  $s_{eff}(c) = 6$ . This alternative is also forced by the need to get mixing of third generation with the lower ones. For baryons same value works best.
- e) Unitarity requirement fixes the values of  $s_{eff}(b)$  and  $s_{eff}(t)$ : one must have  $\sum_i s_{eff}(q_i) = 69$ .
- f) The assumption  $s(u) = 6$  looks at first rather crazy: if other angles are small  $u$  quark would spend approximately time fraction  $3/9$  more time  $g = 1$  state than in  $g = 0$  state! This could however solve the spin crisis of the proton: the fraction of parton spin from the spin seems to be only 30 per cent. Assume that the measured non strange parton spin corresponds to  $g = 0$  spin fraction in proton. If  $ud$  pair is in spin singlet state then its

contribution to spin vanishes and only the contribution of  $d$  remains, which gives approximate spin fraction  $(9 - s(u))/9 = 1/3$  for  $s(u) = 6$ : agreement is quite good! The corrections coming from the mixing with higher generation slightly increase the fraction of  $g = 0$  spin. For neutron corresponding result is  $(9 - s(d))/9 = 5/9$  maximal mixing scenario.

To summarize:

The scenarios  $(n_1(d) = 3, n_2(d) = 8, n_1(u) = 5, n_2(u) = 6)$  and  $(n_1(d) = 4, n_2(d) = 8, n_1(u) = 6, n_2(u) = 6)$  favoured by spin crisis of proton are the best alternatives as far as masses are considered and predict same Cabibbo angle at the limit, when mixings with the third generation is absent. CKM matrix turns out to depend only very weakly on  $n_{eff}(s)$  and  $K - \pi$  mass difference suggests that value  $n_{eff}(s, M) = 9 < 11$ .

quark	$n$	$n(eff)$
d	0	(3) 4
s	9	11
b	60	(55) 54
u	0	(5) 6
c	9	6
t	60	(58) 57

Table 4.3. The scenario  $(n_1(d) = 4, n_2(d) = 11, n_1(u) = 6, n_2(u) = 6)$  is forced by sensible mass spectrum for baryons and and spin crisis of proton. The values of parenthesis correspond to competing scenario, which doesn't however solve the spin crisis and is not consistent with rationality of U and D matrices.

Mass constraints give for the D matrix the following conditions

$$\begin{aligned}
9|D_{12}|^2 + 60|D_{13}|^2 &= n_1(d) \\
9|D_{22}|^2 + 60|D_{23}|^2 &= n_2(d) \\
9|D_{32}|^2 + 60|D_{33}|^2 &= n_3(d) = 69 - n_2(d) - n_1(d)
\end{aligned} \tag{32}$$

The third condition is not independent since the sum of the conditions is identically true by unitarity.

For  $U$  matrix one has similar conditions. The task is to find unitary mixing matrices satisfying these conditions.

### 4.3.3 The general form of U and D matrices

The general form of U and D matrices is taken to be same as the standard parametrization of Kobayashi-Maskawa matrix.

$$\begin{bmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 \exp(i\delta) & c_1 c_2 s_3 - s_2 c_3 \exp(i\delta) \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 \exp(i\delta) & c_1 s_2 s_3 - c_2 c_3 \exp(i\delta) \end{bmatrix}$$

Table 7. CKM type parametrization for U and D matrices.

Similar parametrization applies to  $U$  matrix. One can multiply the rows and columns of  $U$  and  $D$  with constant phases. Only the multiplication of the columns of  $U$  matrix affects KM matrix defined as

$$V = DU^\dagger \quad (33)$$

It is natural to require that the squares of the moduli of  $D_{ij}$  and  $U_{ij}$  are rational numbers of equivalently: the squares of sines and cosines appearing in  $D$  and  $U$  are rational numbers.

The parametrization used guarantees only formally unitary. Mass conditions give constraints on the values of squares of cosines and sines and the requirement that cosines/sines exist as p-adically real numbers is not trivially satisfied.

### 4.3.4 Explicit treatment of mass conditions

The mass condition for d (u) quark gives following constraint between  $s_1$  and  $s_3$

$$s_3^2 = \frac{(n_1 - 9s_1^2)}{51s_1^2} \quad (34)$$

where  $n_1 = 3$  for  $d$  and  $n_1 = 5$  for  $u$ .

The mass condition for  $s$  (c) quark gives following condition

$$\begin{aligned}
c_2^2 X - V &= -c_2 s_2 Y \\
X &= 51((c_1 s_3)^2 - c_3^2) - 9s_1^2 \\
Y &= 102c_1 c_3 s_3 c_{CP} \\
V &= n_2 - 9 - 51c_3^2
\end{aligned} \tag{35}$$

One can reduce this equation to order equation for  $c_2^2$  by taking squares of both sides. The solution of the resulting equation reads as

$$\begin{aligned}
c_2^2 &= -\frac{b}{2} + \epsilon_1 \sqrt{b^2 - 4c} \\
b &= -\frac{Y^2 + 2VX}{X^2 + Y^2} \\
c &= \frac{V^2}{X^2 + Y^2} \\
\epsilon_1 &= \pm 1
\end{aligned} \tag{36}$$

p-Adicity gives nontrivial additional constraint. The argument of square root must be square of a rational number

$$b^2 - 4c = (m/n)^2 \tag{37}$$

This condition in turn gives second order equation for  $Y^2$ , which can be solved and gives

$$Y^2 = -2V(X - V) + \epsilon_2 2V \sqrt{(X - V)^2 + 4\left(\frac{m_1}{n_1}\right)^2} \tag{38}$$

Here  $(m_1/n_1)$  is related to  $m/n$  by  $(m/n) = 2V(m_1/n_1)$ . Additional consistency condition results from the requirement that also the argument of this square root is square of a rational number. The condition reads as

$$(X - V)^2 = \left(\frac{k}{l}\right)^2 - \left(\frac{m_1}{n_1}\right)^2 \tag{39}$$



The condition states that  $X - V$  is a rational number, whose square is difference of two squares of rational numbers. Taking common denominator for all three rational numbers in equation one obtains just the condition defining Pythagorean triangle! This means that the solutions of the equation can be written in the following form

$$\begin{aligned}
X - V &= \epsilon_3 K \Delta(r, s, \epsilon_4) \\
\frac{k}{l} &= K \Delta(r, s, 0) \\
\frac{m}{n} &= K \Delta(r, s, -\epsilon_4) \\
\Delta(r, s, 0) &= (r^2 + s^2) \\
\Delta(r, s, 1) &= r^2 - s^2 \\
\Delta(r, s, -1) &= 2rs \\
Y^2 &= 2V(-\epsilon_3 K \Delta(r, s, \epsilon_4) + \epsilon_2 \Delta(r, s, 0)) \\
c_2^2 &= -\frac{b}{2} + \epsilon_1 K \Delta(r, s, -\epsilon_4)
\end{aligned} \tag{40}$$

where  $r, s$  are the integers defining Pythagorean triangle: recall that one of these integers is even and one odd. The result means that solutions to the mass conditions are labeled by the rational number  $K$  and two integers. The expression for  $Y^2$  must reduce to one of the following forms ( $V < 0$ )

$$\begin{aligned}
Y^2 &= -2VKf \\
f &= 2r^2, 2s^2 \text{ or} \\
f &= (r + s)^2, (r - s)^2
\end{aligned} \tag{41}$$

This means that  $Y^2$  is rational square for  $f = 2r^2, 2s^2$  if  $KV$  is rational square and for  $f = (r \pm s)^2$  if  $2KV$  is rational square.

It should be noticed also that the consistency condition allows rational version of Lorentz group as symmetry group. The quantity  $(k/l)^2 - (m/n)^2$  corresponds to the Minkowskian line element and Lorentz transformations are realized as  $x \rightarrow \gamma(x - \beta y), y \rightarrow \gamma(y - \beta x)$ , where both  $\beta$  and  $\gamma = 1/\sqrt{1 - \beta^2}$  are rational numbers. This in turn implies that  $\beta$  is in one-one-correspondence with Pythagorean triangles! Pythagorean triangles define rational version of 2-dimensional Lorentz group! There is dual Lorentz group

associated with the side of length  $2rs$ . This group leaves invariant the purely nondiagonal form of line element (light cone coordinates and transformations are represented as:  $(r, s) \rightarrow (\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}r, \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}s)$ ). In both cases the transformations in general do not leave  $r$  and  $s$  integers and must be accompanied by a proper scalings.

One can express the angle parameter  $s_1^2$  in terms of parameters  $K, r, s$  by using the expressions of  $X$  and  $V$  in terms of  $s_1^2$

$$s_1^2 = \frac{n_1}{n_1 + n_2 + \epsilon_3 K \Delta(r, s, \epsilon_4)} \quad (42)$$

From the requirement

$$\begin{aligned} s_1^2 &\simeq \frac{n_1}{9} \\ n_1(d) &= 3(4) \\ n_1(u) &= 5(6) \end{aligned} \quad (43)$$

guaranteeing correct value for Cabibbo angle ( $s_{Cab} \simeq s_1(u)c_1(d) - c_1(u)s_1(d)$ ) one obtains (by noticing that  $n_1 + n_2 = 11$  for both  $U$  and  $D$  case)

$$\begin{aligned} K \Delta(r, s, \epsilon_4) &= k - \delta \\ \epsilon_3 &= -1 \\ k &= n_1 + n_2 - 9 \end{aligned} \quad (44)$$

The parameters  $\delta(u)$  and  $\delta(d)$  are small parameters, which parametrize the deviation of mixing matrix from the matrix describing mixing of two lowest generations only.

The expressions of  $s_1$  and  $s_3$  read as

$$\begin{aligned} s_1^2 &= \frac{n_1}{9 + \delta} \\ s_3^2 &= \frac{n_1 + n_2 - 9 - k + \delta}{51} = \frac{\delta}{51} \end{aligned} \quad (45)$$

It should be noticed that  $k < 3$  is forced by the requirement that  $s_3^2$  is positive.

The expression for the cosine of CP breaking angle is given by

$$\begin{aligned}
c_{CP}^2 &= \frac{Y^2}{R_0} \\
Y^2 &= -2V(-k + \delta + K\epsilon_1\Delta(r, s, 0)) \\
R_0 &= \frac{4(u - n_1)(60 - u)(u - 9)}{u} \\
V &= 2n_2 + n_1 - 69 + 2\epsilon_3 K\Delta(r, s, \epsilon_4) \\
u &= n_1 + n_2 - k + \delta = 9 + \delta
\end{aligned} \tag{46}$$

From the expression of  $u$  one find that  $R_0$  is proportional to the small parameter  $\delta$  and therefore  $c_2^2$  becomes very large unless  $R$  is also proportional to  $\delta$  at this limit. This amounts to the following requirements

$$\begin{aligned}
K\Delta(r, s, 0) &= k - \delta_1 \\
\epsilon_1 &= 1
\end{aligned} \tag{47}$$

which brings second small parameter in the theory and one can write

$$\begin{aligned}
c_{CP}^2 &= \frac{2u}{4(u - n_1)(60 - u)}(69 - 2n_2 - n_1 + 2k - 2\delta)\left(\frac{\delta_1}{\delta} - 1\right) \\
u &= n_1 + n_2 - k + \delta = 9 + \delta
\end{aligned} \tag{48}$$

Unitarity requirement poses strong conditions on the ratio of the parameters  $\delta$  and  $\delta_1$ . When the parameters are identical CP breaking angles are vanishing and CKM matrix becomes purely real.

CP breaking angle is always nonvanishing as can be seen from the expression for the sine of the second angle of the Pythagorean triangle

$$\sin(\phi_P) = \frac{\Delta(r, s, \epsilon_4)}{\Delta(r, s, 0)} = \frac{(k + \delta)}{(k + \delta_1)} \tag{49}$$

For small values of  $\delta$  and  $\delta_1$  one of the angles of the triangle is very small. Since the the two sides of Pythagorean triangle are always even and odd numbers respectively it is not possible to construct Pythagorean triangle degenerate to line so that  $\delta \neq \delta_1$  holds always true. This in turn means that CP breaking is a geometric necessity.

The value of  $c_2^2$  can be expressed as

$$\begin{aligned}
c_2^2 &= -\frac{b}{2} + \frac{\delta_2}{2} \\
b &= -\frac{Y^2 + 2VX}{Y^2 + X^2} \\
Y^2 &= -2V(\delta - \delta_1) \\
V &= 2n_2 + n_1 - 69 - k + \delta \\
X &= V - k + \delta = 2n_2 + n_1 - 69 - 2k + 2\delta
\end{aligned} \tag{50}$$

$$K\Delta(r, s, 0) \equiv \delta_2 = K\sqrt{\Delta(r, s, \epsilon_4)^2 - \Delta(r, s, -\epsilon_4)^2} = \sqrt{\delta^2 - \delta_1^2 + 2k(\delta_1 - \delta)} \tag{51}$$

The parameter  $\delta_2$  is clearly not an independent quantity.

The unitary constraints for  $\delta, \delta_i$  can be written in general form as

$$\begin{aligned}
1 &\leq \frac{\delta_1}{\delta} \leq 1 + \frac{4(u - n_1)(60 - u)}{2u(69 - 2n_2 - n_1 - 2k + 2\delta)} \\
0 &\leq \delta_2 \leq 2 + b \\
u &= n_1 + n_2 - k + \delta = 9 + \delta
\end{aligned} \tag{52}$$

where we have not bothered to rewrite the definition of the function  $b$  appearing in the general expression of  $c_2^2$ . For physical solutions  $c_2^2$  should be near unity, which corresponds to the upper unitarity limit  $\delta_2 = 2 + b$ . In fact unitarity forces it to be so. The numerical value of  $-b/2$  is slightly below unity at the limit  $\delta, \delta_1 \rightarrow 0$  since one has

$$-b(\delta, \delta_1 \rightarrow 0) \rightarrow \frac{2(X + k)}{X}$$

$$2 \frac{(69 - 2n_2 - n_1 + k)}{(69 - 2n_2 - n_1 + 2k)} \leq 2 \quad (53)$$

so that  $s_2^2$  lies in rather narrow range  $(0, 1 - \frac{X+k}{X})$ .

#### 4.3.5 Constraints on U and D matrices from empirical information on CKM matrix

The most recent experimental information [Buras] concerning CKM matrix elements is summarized in table below

$ V(1, 3)  \equiv  V_{ub}  = (0.087 \pm 0.075)V_{cb} : 0.42 \cdot 10^{-3} <  V_{ub}  < 6.98 \cdot 10^{-3}$
$ V(2, 3)  \equiv  V_{cb}  = (41.2 \pm 4.5) \cdot 10^{-3}$
$ V(3, 1)  \equiv  V_{td}  = (9.6 \pm 0.9) \cdot 10^{-3}$
$ V(3, 2)  \equiv  V_{ts}  = (40.2 \pm 4.4) \cdot 10^{-3}$
$s_{Cab} = 0.226 \pm 0.002$

Table 4.4 The experimental constraints on the absolute values of the KM matrix elements.

The expression for Cabibbo angle given by

$$\begin{aligned} \sin(\theta_C) &\equiv \sqrt{1 - |V_{11}|^2} \\ V_{11} &= c_1(u)c_1(d) + s_1(u)s_1(d)(c_3(u)c_3(d) + s_3(u)s_3(d)) \\ &= c_1(u)c_1(d) + s_1(u)s_1(d)\cos(\theta_3(u) - \theta_3(d)) \\ &= \cos(\theta_1(u) - \theta_1(d)) - s_1(u)s_1(d)(1 - \cos(\theta_3(u) - \theta_3(d))) \end{aligned} \quad (54)$$

gives for  $\theta_3(u) - \theta_3(d) = 0$

$$\sin(\theta_c) = \frac{(\sqrt{n_1(u)}\sqrt{9 - n_1(d)} - \sqrt{n_1(d)}\sqrt{9 - n_1(u)})}{\sqrt{(9 + \delta(d))(9 + \delta(u))}} \quad (55)$$

The requirement that the value is sufficiently near to the observed value  $\sin(\theta_c) \simeq 0.226 \pm 0.002$  gives constraints on the values  $n_i(u/d)$  and  $\delta(u/d)$ .

a) ( $n_1(d) = 3, n_2(u) = 5$ ) and ( $n_1(d) = 4, n_1(u) = 6$ ) yield same value at the 0.2236 at small  $\delta$  limit, which is just at the lower bound of experimental

uncertainties.

b) Nonzero value for  $\theta_3(u) - \theta_3(d)$  however tends to increase the value of Cabibbo angle. If  $s_3(d)$  and  $s_3(u)$  are of same sign maximum of  $s_{Cab}$  is achieved for  $\delta(u) = 0$  (  $U$  matrix is nontrivial for two lowest generations only).

c) For  $(n_1(d) = 4, n_1(u) = 6)$  scenario the maximum value  $s_{Cab} \simeq 0.22601$  (accidentally same as experimental mean value!) of Cabibbo angle and is achieved at  $\delta(d) \simeq 0.032601$ .

d) For  $(n_1(d) = 3, n_1(u) = 5)$  scenario maximum  $s_{Cab} \simeq 0.2255$  is achieved at  $\delta(d) \simeq 0.0031976$ . It turns out that  $\delta(u) = 0$  implies too large values for the elements  $V(1, 3)$  and  $V(3, 1)$ . This decreases the prediction for Cabibbo angle so that the scenario  $(n_1(d) = 4, n_1(u) = 6)$  is slightly favoured.

e) The alternative  $(n_1(d) = 2, n_1(u) = 4)$  yields at  $\delta = 0$  limit value larger than experimental value and by choosing appropriately the values of  $\delta$  it is easy to reproduce the experimental value of Cabibbo angle. The values of  $\delta$  must be of order  $1/2$  and are suspiciously large.

The second constraint comes from smallness of  $V(1, 3)$  and  $V(3, 1)$ . From the table above it is clear that the moduli of the elements  $V_{31} = V_{td}$  and  $V_{13} = V_{ub}$  should be below  $10^{-2}$ . These elements correspond to the inner products of first and third rows for  $U$  and  $D$  type matrices. By writing these inner products explicitly one finds for the imaginary part the expression

$$\begin{aligned} \text{Im}(V(1, 3)) &= c_2(u)s_1(d)\sin(\theta_3(u) - \theta_3(d))s_{CP}(u) \\ \text{Im}(V(1, 3)) &= -c_2(d)s_1(u)\sin(\theta_3(u) - \theta_3(d))s_{CP}(d) \end{aligned} \quad (56)$$

The values of these quantities are small if  $s_{CP}(d)$ ,  $s_{CP}(u)$  and/or  $\theta_3(u) - \theta_3(d)$  are small enough.  $\theta_3(u) - \theta_3(d)$  is restricted from below by the requirement that Cabibbo angle is within experimental uncertainties.

The real parts for  $V_{13}$  and  $V_{31}$  can be written as

$$\begin{aligned} \text{Re}(V(1, 3)) &= -c_1(d)(s_1s_2)(u) + (s_1c_3)(d)(c_1s_2c_3 + c_2s_3c_{CP})(u) \\ &\quad + (s_1s_3)(d)(c_1s_2s_3 - c_2c_3c_{CP})(u) \\ \text{Re}(V(3, 1)) &= -c_1(u)(s_1s_2)(d) + (s_1c_3)(u)(c_1s_2c_3 + c_2s_3c_{CP})(d) \end{aligned}$$

$$+ (s_1 s_3)(u)(c_1 s_2 s_3 - c_2 c_3 c_{CP})(d) \quad (57)$$

In order to minimize the values of  $V(1, 3)$  and  $V(3, 1)$  one can vary the values of  $\delta_1(u)$ ,  $\delta_1(d)$  and choose the sign for  $s_2$ ,  $c_2$  freely. Numerical experimentation shows that the correct manner to achieve minimization is to assume

$$\begin{aligned} s_2(u), s_2(d) & \text{ small} \\ c_2(d) & > 0, c_2(u) < 0 \end{aligned} \quad (58)$$

The experimental bounds for the moduli of CKM matrix element give good estimates for the parameters  $s_1, s_2, s_3$  appearing in the CKM matrix.

$$\begin{aligned} s_1 &= .226 \pm .002 \\ s_1 s_2 &= V(3, 1) = (9.6 \pm .9) \cdot 10^{-3} \\ s_1 s_3 &= V(1, 3) = (.0087 \pm .075) \cdot V(2, 3) \\ V(2, 3) &= (40.2 \pm 4.4) \cdot 10^{-3} \end{aligned} \quad (59)$$

The remaining parameter is  $\sin(\delta)$  or equivalently the CP breaking parameter  $J$

$$J = \text{Im}(V(1, 1)V(2, 2)\bar{V}(1, 2)\bar{V}(2, 1)) = c_1 c_2 c_3 s_2 s_3 s_1^2 \sin(\delta) \leq 6.7 \cdot 10^{-5} \quad (60)$$

where the upper bound is for  $\sin(\delta) = 1$  and the previous average values of the parameters  $s_i, c_i$  (note that the poor knowledge of  $s_3$  affects on the upper bound for  $J$  considerably). Information about the value of  $\sin(\delta)$  as well as on the range of possible top quark masses comes from CP breaking in  $K - \bar{K}$  and  $B - \bar{B}$  systems.

The observables in  $K_L \rightarrow 2\pi$  system [Paschos and Turke]

$$\eta_{+-} = \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} = \epsilon + \frac{\epsilon'}{1 + \omega/\sqrt{2}}$$

$$\begin{aligned}
\eta_{00} &= \frac{A(K_L \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)} = \epsilon - 2 \frac{\epsilon'}{1 - \sqrt{2}\omega} \\
\omega &\sim \frac{1}{20} \\
\epsilon &= (2.27 \pm .02) \cdot 10^{-3} \cdot \exp(i43.7^\circ) \\
|\frac{\epsilon'}{\epsilon}| &= (3.3 \pm 1.1) \cdot 10^{-3}
\end{aligned} \tag{61}$$

The phases of  $\epsilon$  and  $\epsilon'$  are in good approximation identical. CP breaking in  $K - \bar{K}$  mass matrix comes from the CP breaking imaginary part of  $\bar{s}d \rightarrow s\bar{d}$  amplitude  $M_{12}$  (via the decay to intermediate  $W^+W^-$  pair) whereas  $K^0 \bar{K}^0$  mass difference  $\Delta m$  comes from the real part of this amplitude: the calculation of the real part cannot be done reliably for kaon since perturbative QCD does not work in the energy region in question. One can however relate the real part to the known mass difference between  $K_L$  and  $K_S$ :  $2\text{Re}(M_{12}) = \Delta m$ .

Using the results of [Paschos and Turke]) one can express  $\epsilon$  and  $\epsilon'/\epsilon$  in the following numerical form

$$\begin{aligned}
|\epsilon| &= \frac{1}{\sqrt{2}} \frac{\text{Im}(M_{12}^{sd})}{\Delta m_K} - .05 \cdot |\frac{\epsilon'}{\epsilon}| = 2J(22.2B_K \cdot X(m_t) - 5.6B'_K) \\
|\frac{\epsilon'}{\epsilon}| &= 2J \cdot 5.6B'_K \\
X(m_t) &= \frac{H(m_t)}{H(m_t = 60 \text{ GeV})} \\
H(m_t) &= -\eta_1 F(x_c) + \eta_2 F(x_t)K + \eta_3 G(x_c, x_t) \\
x_q &= \frac{m(q)^2}{m_W^2} \\
K &= s_2^2 + s_2 s_3 \cos(\delta) \\
\eta_1 &\simeq 0.7 \quad \eta_2 \simeq 0.6 \quad \eta_3 \simeq 0.5
\end{aligned} \tag{62}$$

Here the values of QCD parameters  $\eta_i$  depend on top mass slightly and are given for  $m_t = 60 \text{ GeV}$ :  $\eta_i$  as well as parameters appearing in the explicit expression of  $\epsilon'/\epsilon$  are assumed to be same for higher top masses in the following estimates.  $B'_K$  and  $B_K$  are strong interaction matrix elements and



vary between  $1/3$  and  $1$ . The functions  $F$  and  $G$  [Paschos and Turke]. are given by

$$\begin{aligned} F(x) &= x\left(\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right) + \frac{3}{2}\left(\frac{x}{x-1}\right)^3 \ln(x) \\ G(x, y) &= xy\left(\frac{1}{x-y}\left(\frac{1}{4} + \frac{3}{2}\frac{1}{1-x} - \frac{3}{4}\frac{1}{(1-x)^2}\right)\ln(x) + (y \rightarrow x) - \frac{3}{4}\frac{1}{(1-x)(1-y)}\right) \end{aligned} \quad (63)$$

One can solve parameter  $B'_K$  by requiring that the value of  $\epsilon'/\epsilon$  corresponds to the experimental mean value:

$$B'_K = \frac{.05}{11.2 \cdot J} \frac{\epsilon'}{\epsilon} \quad (64)$$

One can also solve the parameter  $J$  and  $\sin(\delta)$  in terms of  $\epsilon$  for a given top mass. Despite the poor knowledge of  $V(1, 3)$  one can make the following conclusions:

a) The requirement that  $B'_K$  belongs to the range  $(1/3, 1)$  limits the values of CP invariant  $J$  considerably:

$$1.5 \cdot 10^{-5} \leq J \leq 4.4 \cdot 10^{-5} \quad (65)$$

b) The requirement that  $\sin(\delta)$  is nonnegative drops  $k = 89$  top from consideration: the point is that for large top mass the sign of the function  $H(m_t)$  changes and this implies that the relative sign of  $\epsilon$  and  $\epsilon'$  changes. Also the absolute value of  $B'_K$  becomes larger than one.

c) For  $k = 97$  top quark one has  $\sin(\delta) > 1$ .

d) For the observed top quark candidate one can reproduce all values of  $\sin(\delta)$  in the allowed range for  $J$  so that the precise value of the CP breaking parameter remains open.

A second source of information comes from  $B - \bar{B}$  mass difference. At the energies in question perturbative QCD is expected to be applicable for the calculation of the mass difference and mass difference is predicted correctly if the mass of the top quark is essentially the mass of the observed top

candidate [Ali and London]. It seems that top quark cannot correspond to neither to  $k = 89$  nor  $k = 97$  condensation level. A possible explanation for the discrepancy is that top quark corresponds to quantum mechanical mixture of these levels and that the mass squared is therefore expectation value: if top quark spends about 4 percent of its time on  $k = 89$  level the mass is the observed mass.

Physically acceptable CKM matrices can be found numerically by varying  $\delta(u)$  and  $\delta_1(d), \delta_1(u)$  inside unitary limits. CKM matrix is not sensitive to the values of  $n_2(d)$  and  $n_2(u)$  for the physical parameter values. The table below provides an example of ( $n_1(d) = 4, n_1(u) = 6$ ) CKM matrix for which the values of matrix elements are within the experimental bounds. The value of Cabibbo angle is  $s_{Cab} = 0.22469$ . The values of  $s_2^2(d)$  and  $s_2(u)$  are rather small. The value of CP-breaking parameter  $J = \text{Im}(V_{11}V_{22}\bar{V}_{12}\bar{V}_{21})$  is  $J \simeq 4.986 \cdot 10^{-5}$  quite near to the upper bound deduced above. It must be stressed that the example is not probably the best one. For example, by suitably varying the parameters one could make  $V_{td}$  smaller.

V	d	s	b
u	9.7443 e-1	2.2459e-1-1.7011e-4i	-1.5646e-3+ 6.4272e-3i
c	2.2445e-1 -8.0600e-5i	-9.7368e-1-7.2035e-5i	3.9319e-02 - 4.8791e-3i
t	5.1355e-3 +9.0110e-3i	-3.6465e-2-1.3277e-2i	-9.6465e-1-2.6047e-1i
V	d	s	b
u	9.7443e-1	2.2459e-1	.66149e-3
c	2.2445e-1	9.7368e-1	3.9621e-2
t	1.0372e-2	3.8807e-2	9.9919e-1

Table 4.5. Example of CKM matrix satisfying experimental constraints [Buras] with parameters  $n_1(d) = 4, n_2(d) = 11, n_1(u) = 6, n_2(u) = 6, \delta(d) = 0.0318, s_2^2(d) = 8 \cdot 10^{-5}, \delta(u) = 0.009, s_2^2(u) = 7 \cdot 10^{-4}$ . The value of CP breaking invariant  $J$  is  $J = 4.986 \cdot 10^{-5}$ .

#### 4.3.6 Number theoretic conditions on U and D matrices

CKM matrix represents together with quark and lepton masses the basic parameters of the standard model. It would be nice if p-adic approach would predict only the masses but also U and D matrices uniquely. The p-adic

and real probability concepts are formally equivalent if the moduli squared of U,D and CKM matrices are rational numbers. This requirement indeed poses strong number theoretical conditions on U and D matrices and gives hopes of getting unique CKM matrix.

Rational moduli squared are obtained if the elements of the U and D matrices are rational numbers. This implies that the angles appearing in the matrices correspond to Pythagorean triangles. It is not however clear whether the Pythagorean conditions can be satisfied. This forces to look carefully for the general structure of U,D and CKM matrices. The following observations emerge.

- a) In p-adic regime, what might be called irreducible phases, are possible. For instance,  $x = \sqrt{5}$  does not exist as rational number but one can replace it with rational quantity  $2 + i$  having irreducible phase, which cannot be cancelled away in the realm of rationals. Also irrational phases are possible:  $\sqrt{3} \rightarrow \sqrt{2} + i$  gives simple example of this phenomenon. If one can replace ordinary square roots with square roots containing irreducible phase, which is constant along rows of U(D) matrix one can perhaps obtain unitary and rational U (D) matrix. Even irrational irreducible phases are possible if only moduli squared are required to be rational.
- b) The study of U (D) matrix shows that  $c_1, s_1$  and  $c_{CP}, s_{CP}$  cannot have irreducible phase: the reason is that these quantities do not appear homogeneously on the rows of  $U/D$  matrix. It also turns out that the Pythagorean conditions do not lead to any contradictions.
- c) The parameters  $s_2, c_2$  can be given common irreducible phase without affecting the rationality of the moduli squared or spoiling unitarity conditions. The phase cancels also from the moduli squared of CKM matrix elements. A nice manner to get common irreducible rational phase is to assume that these angles correspond to triangle with integer valued shorter sides and to require that the square root  $\sqrt{X^2 + Y^2}$  of their common denominator  $\sqrt{X^2 + Y^2}$  is replaced with  $X + iY$ . Number theoretic constraints come from the requirement that  $\frac{s_2^2}{c_2^2}$  is square of a rational number.
- d) Also  $s_3, c_3$  can be given same phase same for all rows. U and D matrix unitarity remain unaffected and probabilities rational. This phase must cancel from KM matrix and this requires that the phase is identical for D and U matrices. The general form of  $s_3^2 = \frac{\delta}{51} = \frac{km}{kn51}$  suggests that the possible phase comes from the denominator  $kn51$ :  $\sqrt{kn51} \rightarrow a + ib$ . The phase can be

chosen to be rational and  $s_3, c_3$  correspond to a triangle with integer valued shorter sides. Pythagorean triangle is possible for the scenario solving proton spin crisis, only. The phases associated with U and D matrices are identical if  $k(U)n(U)/k(D)n(D)$  is square of a rational number.

To sum up, the most general scenario allowing rational probabilities satisfies following conditions

$$\begin{array}{ll}
s_1, c_1 & \textit{Pythagorean} \\
s_{CP}, c_{CP} & \textit{Pythagorean} \\
\frac{s_2}{c_2} & \textit{rational} \\
\frac{s_3}{c_3} & \textit{rational} \\
\frac{n(U)k(U)}{n(D)k(D)} = \left(\frac{k}{l}\right)^2 & (66)
\end{array}$$

For rational U,D and CKM matrices all these angles are Pythagorean. In the following the number theoretical consistency of these conditions is studied. The method is to write various conditions modulo 8 (modulo n for arbitrary n gives valuable information). The reason is that the square of an odd number is always one modulo 8. In particular, the squares for the sides of Pythagorean triangle are equal to 1 and 0 modulo 8 so that modulo 8 equations are easily derivable. The ratio  $s^2/c^2$  for Pythagorean angles ( $s_1, c_1$ ) and  $s_{CP}, c_{CP}$ ) imply congruence relating the parameter  $K$  to the parameters  $r, s$  and integers  $n_i, k$ . Similar congruences are obtained from the requirement that  $s_2/c_2$  is rational for  $s_2, c_2$  and  $s_3, c_3$ . Four independent nonlinear congruences are obtained as consistency conditions for U and D matrix so that these matrices might be even unique!

### 1. Pythagorean conditions force the scenario solving proton spin crisis

The most stringent requirement on U and D matrix is that even the sines and cosines are rational numbers and therefore correspond to Pythagorean squares ( $r, s$ ) with sides  $r^2 - s^2$  and  $2rs$  ( $r$  or  $s$  are not both even or odd) and sines and cosines are given by the expressions

$$\begin{aligned}
\sin(u) &= \frac{2rs}{(r^2 + s^2)} \\
\cos(u) &= \frac{(r^2 - s^2)}{(r^2 + s^2)}
\end{aligned} \tag{67}$$

or by the expression obtained by the exchange  $2rs \leftrightarrow r^2 - s^2$ .

To see whether the mass conditions allow all angle parameteres to be Pythagorean angles one can write the equations modulo 8 and use the elementary fact the square of odd number is always one modulo 8 whereas the square of even number is always 4 or 0 modulo 8. As a consequence one has

$$\begin{aligned}
(r^2 + s^2)^2 = (r^2 - s^2)^2 &= 1 \mod 8 \\
(2rs)^2 &= 0 \mod 8
\end{aligned} \tag{68}$$

The equations for  $D$ , when written modulo 8 give

$$\begin{aligned}
|D_{12}|^2 + 4|U_{13}|^2 &= n_1(d) \\
|C_{22}|^2 + 4|D_{23}|^2 &= n_2(d)
\end{aligned} \tag{69}$$

Same applies to  $U$  matrix. By writing the left hand side of the equations explicetly in terms of the parametrization one finds that the first term on the left hand side of the equations is sum of one or two terms with value equal to 0 or 1 and therefore can have only the values 0, 1, 2. This implies that left hand side can have value in the set  $n = 0, 1, 2, 4, 5, 6, 7$ , which does not contain  $n = 3$ . Therefore  $n_1(d) = 3$  is impossible to realize using rational  $D$  matrix whereas  $n_1(d) = 4$  favoured by proton spin crisis could allow rational  $U$  and  $D$  matrices.

## 2. Pythagorean conditions for $(s_1, c_1)$

The Pythagreaon conditions for  $(s_1, c_1)$  read as

$$s_1^2 = \frac{\Delta(r_1, s_1, \epsilon(1))}{k_1}$$

$$c_1^2 = \frac{\Delta(r_1, s_1, -\epsilon(1))}{k_1} \quad (70)$$

$$(71)$$

where the functions  $\Delta$  are already familiar. Using the expression  $s_2^2 = n_1/(9+\delta)$  one finds that the Pythagorean triangle condition for  $s_1^2$  gives the following possibilities

$n_1 k_1$	$((9 - n_1)n + m)k_1$	$(9n + m)k_1$
$(2r_1 s_1)^2$	$(r_1^2 - s_1^2)^2$	$(r_1^2 + s_1^2)^2$
$(r_1^2 - s_1^2)^2$	$(2r_1 s_1)^2$	$(r_1^2 + s_1^2)^2$

Table 4.6. Various alternative solutions to Pythagorean conditions for  $(s_1, c_1)$

Here  $k_1$  is an rational number resulting from the nonuniqueness of the representations  $\delta = m/n$ . There are two cases corresponding to odd and even  $n_1$ . The condition  $n_1 n k_1 = 1 \pmod{8}$  holds true as difference of sides squared for Pythagorean triangle. The condition implies is satisfied if  $n_1 k_1 = 1 \pmod{8}$  and  $n = 1 \pmod{8}$  holds true, which leaves only the second alternative for the sides of Pythagorean triangle. This in turn implies  $(9n + m)k_1 = 1 \pmod{8}$ , which requires  $9n + m$  to be divisible by  $n_1$  and  $(9n + m)/n_1 = 1 \pmod{8}$ . This gives

$$\begin{aligned} n &= 1 + 8p \\ m &= n_1 - 1 + (qn_1 - 9p - 1)8 \\ \delta &= \frac{n_1 - 1 + (qn_1 - 9p - 1)8}{1 + 8p} \end{aligned} \quad (72)$$

The two conditions for  $\delta$  do not lead to any obvious contradictions.

Pythagorean condition allow to solve  $K$  in terms of  $(r, s, r_1, s_1)$  from the ratio  $s_1^2/c_1^2$ , which does not contain  $k_1$ .

$$K = n_1 - 9 + n_1 \frac{\Delta^2(r_1, s_1, -\epsilon(1))}{\Delta^2(r_1, s_1, \epsilon(1))}. \quad (73)$$

Later an analogous formula will be derived from  $s_{CP}, c_{CP}$  Pythagorean conditions and this gives rise to consistency condition between various integers.

### 3. $s_3, c_3$ can be Pythagorean

The moduli squared of U and D matrix elements are rational if the shorter sides of the  $s_3, c_3$  triangle can be chosen to be integers. Here it will be shown that the conditions for rational phase for  $s_3, c_3$  are in accordance with the condition  $n = 1 \bmod 8$  derived from  $s_1, c_1$  conditions and that also Pythagorean alternative is possible. For  $s_3^2 = \delta/51$  and  $c_3^2$  the rational irreducible phase conditions imply that these angles correspond to a triangle with integer valued shorter sides  $k_3, l_3$  and the possible rational phase can be chosen to be  $k_3 + il_3$

$$\begin{aligned} \delta &= \frac{km}{kn} \\ 51kn &= k_3^2 + l_3^2 \\ 51kn &= m^2 + r^2 \end{aligned} \quad (74)$$

The condition  $51kn = k_3^2 + l_3^2 \bmod 8$  together with  $n = 1 \bmod 8$  implies  $3k = k_3^2 + l_3^2 \bmod 8$ , which gives the following possibilities for  $k$  and  $k_3$  and  $l_3$

$k \bmod 8$	0	3	6	4	7
$k_3^2 + l_3^2$	0	1	2	4	5

Table 4.7. The values of  $k \bmod 8$  for various combinations of  $k_3, l_3$ .

The ratio  $s_3/c_3 = (k_3/l_3)$  is rational and this gives linear congruence relating  $K$  to the parameters  $(r, s)$  and rational number

$$\begin{aligned}
K &= \frac{(51\beta_3^2 - k(1 + \beta_3^2))}{\Delta(r, s, \epsilon_4)(1 + \beta_3^2)} \\
\beta_3 &= \frac{k_3}{l_3}
\end{aligned} \tag{75}$$

If  $s_3, c_3$  corresponds to Pythagorean triangle the additional condition  $\beta = (r^2 - s^2)/2rs$  or  $\beta = 2rs/(r^2 - s^2)$  holds true.

The requirement that the possible irreducible phases associated with  $U$  and  $D$  matrices are identical implies the condition

$$\frac{k(U)n(U)}{k(D)n(D)} = \gamma^2 \tag{76}$$

where  $\gamma$  is rational number. For Pythagorean case this condition is not needed.

#### 4. Pythagorean conditions for $c_{CP}, s_{CP}$

$c_3, s_3$  should be rational numbers in order to get rational moduli squared for  $U, D$  and CKM matrices. The general expression for  $c_{CP}^2$  reads as

$$\begin{aligned}
c_{CP}^2 &= \frac{Y^2}{R_0} \\
Y^2 &= 2(72 - 2n_2 - n_1 - 2\delta)(\delta_1 - \delta) \\
R_0 &= 4(51c_1 \text{abs}(c_3 s_3))^2 = \frac{4(u - n_1)(60 - u)(u - 9)}{u} \\
u &= 9 + \delta \\
\delta &= \frac{n_1 - 1 + (qn_1 - 9p - 1)8}{1 + 8p}
\end{aligned} \tag{77}$$

$R_0$  contains only  $\text{abs}(c_3 s_3)$  term so that p-adically rational square root exists for  $R_0$ . For  $Y^2$  square root exists also provided the function  $-2Vf$  is rational square (  $Y^2 = -2VKf$  with  $f = 2r^2, 2s^2$  or  $f = (r \pm s)^2$ ). Therefore only



the rationality condition for  $s_{CP}$  can turn out troublesome. The common factor 2 drops from the ratio defining  $c_{CP}^2$ .

The Pythagorean conditions read in present case as

$$\begin{aligned} Y^2 k_{CP} &= (r_{CP}^2 - s_{CP}^2)^2 \\ R_0 k_{CP} &= (2r_{CP}s_{CP})^2 \end{aligned} \quad (78)$$

Also the second alternative obtained by permuting the sides of the triangle turns out to be excluded.  $k_{CP}$  is essentially the common denominator of rational numbers involved.

Again one can write the conditions modulo 8 in order to derive consistency conditions. The general form for  $Y^2$  and  $R_0$  modulo 8 are

$$\begin{aligned} Y^2/2 &= -((2(n_1 + n_2) + 3)(-\delta_1 - n_1 + 1) = -(5 + 2k)(-\delta_1 - n_1 + 1) \\ R_0/2 &= \frac{32(4 - n_1)(n_1 - 1)}{n_1} \\ \frac{Y^2}{R_0} &= \frac{-(5 + 2k)(-\delta_1 - n_1 + 1)n_1}{32(4 - n_1)(n_1 - 1)} \\ k &= n_1 + n_2 - 9 \end{aligned} \quad (79)$$

From this expression it is clear that for even  $n_1$  one can always divide  $R_0$  by  $2n_1$  and still have  $R_0/2n_1 = 0 \mod 8$  for interesting values of even  $n_1$ . The division with  $n_1$  implies the choice  $k_{CP} = k_B/2n_1$  in general so that one has

$$Y^2 k_{CP} = (5 + 2k)(-\delta_1 + n_1 - 1)k_B = 1 \mod 8 \quad (80)$$

Writing  $\delta_1 = p_1/q_1$  and taking  $k_B = q_1$  one has

$$\begin{aligned} -p_1 + (n_1 - 1)q_1 &= s(k) \mod 8 \\ s(k \equiv n_1 + n_2 - 9 = 2) &= 1 \mod 8 \\ s(k = 1) &= 7 \mod 8 \end{aligned} \quad (81)$$

This condition does not lead to no obvious contradiction with earlier results.

The explicit expression of  $Y^2$  and  $R_0$  allows to write the corresponding Pythagorean conditions in explicit form

$$\begin{aligned}
2(69 - 2n_2 - n_1 + K\Delta(r, s, \epsilon_4))Kf &= (r_{CP}^2 - s_{CP}^2) \\
\frac{4(u - n_1)(60 - u)(u - 9)}{u}k_{CP} &= (2r_{CP}s_{CP})^2 \\
f &= 2r^2, 2s^2 \text{ or } (r \pm s)^2 \\
u &= n_1 + n_2 - K\Delta(r, s, \epsilon_4) \quad (82)
\end{aligned}$$

Taking the ratio of the conditions one obtains equations given  $K$  in terms of the integers associated with two Pythagorean triangles and integers  $n_i, k$ .

$$\begin{aligned}
2(69 - 2n_2 - n_1 + K\Delta(r, s, \epsilon_4))Kf(r_{CP}s_{CP})^2u & \\
= (u - n_1)(60 - u)(u - 9)(r_{CP}^2 - s_{CP}^2) & \\
u &= n_1 + n_2 - K\Delta(r, s, \epsilon_4) \quad (83)
\end{aligned}$$

The resulting equation is third order in  $K$  and one must require that the solutions are rational, which gives additional conditions for the integers  $r, s$  and  $r_{CP}, s_{CP}$  appearing in the equation. One can also substitute the expression for  $K$  obtained from the Pythagorean conditions for  $s_1$  to this equation and get nonlinear congruence as a consistency condition relation for the integers  $r, s, r_1, s_1, r_{CP}, s_{CP}$ .

## 5. Irreducible phase conditions for $c_2, s_2$

The remaining number theoretic conditions come from  $c_2^2$  and  $s_2^2$ . There are two possible manners to obtain rational probabilities.

a)  $c_2$  and  $s_2$  are rational numbers (Pythagorean triangle) so that phases are trivial.

b)  $c_2$  and  $s_2$  allow identical irreducible rational phases coming from the common denominator of the quantities defining these parameters.

The analysis of consistency conditions proceeds by writing  $c_2^2/s_2^2$  as rational square and by looking for the consequences. Also modulo 8 analysis is possible.

The expressions for  $c_2^2$  and  $s_2^2$  read as

$$\begin{aligned}
s_2^2 &= \frac{Y^2(1 - \delta_2) + X(-2V + X(2 - \delta_2))}{2(Y^2 + X^2)} \\
c_2^2 &= \frac{Y^2(1 + \delta_2) + X(2V + X\delta_2)}{2(Y^2 + X^2)} \\
X &= 2n_2 + n_1 - 69 - 2k + 2\delta \\
V &= X + k - \delta
\end{aligned} \tag{84}$$

The obvious manner to get irreducible phase is to perform the replacement  $X^2 + Y^2 \rightarrow X + iY$ . One must however be careful since  $Y^2$  and  $X^2$  are only formally squares and it might be necessary to introduce rational multiplier for  $X^2 + Y^2$  to get irreducible phase.

The requirement is that the numerators of  $s_2^2$  and  $c_2^2$  are rational numbers implies that the ratio  $c_2^2/s_2^2$  is square of a rational number:

$$\begin{aligned}
\frac{Y^2(1 + \delta_2) - X(2V + X\delta_2)}{Y^2(1 - \delta_2) + X(2V + X(2 - \delta_2))} &= \alpha^2 \\
\alpha &= \frac{k_2}{l_2}
\end{aligned} \tag{85}$$

Here the rational number  $\alpha$  is restricted to rather narrow range by unitary. This gives second order equation for  $X$  with consistency conditions resulting from the requirement that the argument of the square root appearing in the solution is square of a rational number.

$$\begin{aligned}
X &= -\frac{b}{2} - \sqrt{b^2 - 4c} \\
b &= 2 \frac{((\delta_1 - \delta)((1 + \delta_2) - (1 - \delta_2)\alpha^2) + (k - \delta_2)(1 + \alpha^2))}{(2 + \delta_2)(1 + \alpha^2)} \\
c &= -\frac{2(\delta_1 - \delta)(k - \delta)(1 - \alpha^2 + \delta_2(1 + \alpha^2))}{(2 + \delta_2)(1 + \alpha^2)} \\
\alpha &= \frac{k_2}{l_2}
\end{aligned} \tag{86}$$

The consistency condition for this equation is that  $b^2 - 4c$  is square of a rational number.

$$\begin{aligned} b^2 - 4c &= \beta^2 \\ \beta &= \frac{k_3}{l_3} \end{aligned} \tag{87}$$

For Pythagorean triangle one must replace  $\beta$  with the ratio of shorte sides of Pythagorean triangle. It is possible to perform modulo 8 analysis for this condition using the information about  $\delta$  and  $\delta_1$  (recall that  $\delta_2$  is expressible in terms of  $\delta_1$  and  $\delta$ ). In particular, one can feed in the modulo 8 information about  $\delta$  and  $\delta_1$ .

Since the quantities  $\delta, \delta_1, \delta_2$  are all defined in terms of the parameters  $K, r, s$  defining the basic Pythagorean triangle the consistency condition gives condition for these three numbers plus integers  $n_i$ . Using the definitions

$$\begin{aligned} K\Delta(r, s, \epsilon_4) &= k - \delta \\ K\Delta(r, s, 0) &= k + \delta_1 \\ K\Delta(r, s, -\epsilon_4) &= \delta_2 \end{aligned} \tag{88}$$

one can transform this condition to fourth order equation for  $K$ . One manner to proceed is to solve this consistency condition for  $K$ . The rationality requirement for the solution can be used also now to derive constraints on the integers  $r, s$  of allowed Pythagorean triangles. A second manner to proceed is to substitute the expression of  $K$  in terms of  $r_1, s_1, r, s$  to the equation and solve the resulting nonlinear congruence. The requirement that integer solution is obtained is quite restrictive.

#### 4.3.7 Summary

It is useful to collect the results found from the number theoretic analysis.

a) In principle it is possible to obtain U,D and CKM matrices defining rational probabilities by allowing common rational irreducible phases for  $s_2, c_2$  and  $s_3, c_3$  respectively whereas  $s_1$  and  $c_1$  and  $s_{CP}, c_{CP}$  are rational numbers and correspond to certain Pythagorean triangles. U and D matrices can

be also rational for the scenario solving proton spin crisis so that all angles correspond to Pythagorean triangles. Rational  $su(3)$  matrices form a group and probably the mathematical literature contains explicit characterization of the matrix elements: it might be easy to see how uniquely the two constraints for mass squared expectation values fix the physically allowed  $U$  and  $D$  matrices.

c) The requirement that the tangents associated with the four angle parameters are rational implies four consistency conditions allowing to express the parameter  $K$  as a solution of, in general nonlinear, rational congruence. Thus there are altogether 4 different equations for the parameter  $K$  and these must be mutually consistent! If Pythagorean triangles are in question the expressions for the tangents of the angles must satisfy additional conditions. That only few solutions to congruences exist is suggested by classical result of number theory [Chowia]: the congruence  $P(x, y) = c$ , where  $P$  is homogeneous polynomial of  $x, y$  with integer coefficients of degree  $n \geq 3$  and  $P(x, 1)$  is irreducible in field of rationals has only finite number of integer solutions.

d) There is also a consistency condition relating  $U$  and  $D$  matrices, which comes from the requirement that the irreducible phases associated with  $c_3, s_3$  are identical for  $U$  and  $D$ .

d) Unitarity gives bounds on the parameters  $\delta, \delta_1, \delta_2$ : in particular  $s_2$  varies in rather narrow range  $(0, 1 - (X + k)/X)$  as a consequence. The only possibly Pythagorean alternative is  $(n_1(d) = 4, n_2(d) = 11, n_1(u) = 6, n_2(u) = 6)$  scenario, which provides good understanding of hadron masses, allows identical mixing matrices for baryonic and mesonic quarks, allows nonvanishing first order color binding energy also for baryons and provides a possible solution to the proton spin crisis.  $K - \pi$  mass difference favours  $n_2(d, M) = 11 < n_2(d, B) = 13$  for mesons: CKM matrix is essentially identical with the baryonic one also with this choice. CP breaking is a geometric necessity resulting from the impossibility of Pythagorean triangle with one side having zero length. For a suitable choice of free parameters the elements of CKM matrix are within experimental bounds and CP breaking parameter  $\epsilon$  has correct order of magnitude.

#### 4.4 Color Coulombic interaction, color magnetic hyperfine splitting and isospin-isospin interaction

Color coulombic interaction gives contribution, which can be in principle different for each quark pair since the average quark distances need not be identical so that one has

$$\Delta s_c = \sum_{pairs} c(q(i), q(j)) \quad (89)$$

where the elements  $c(q(i), q(j))$  are negative integers. In the sequel the elements are assumed to be constant for each quark pair but different for baryonic and mesonic quarks.

Color magnetic hyperfine splitting makes it possible to understand the  $\pi - \rho, K - K^*, N - \Delta$ , etc. mass differences [Close]. The interaction energy is of form

$$\Delta E = S \sum_{pairs} \frac{\bar{s}_i \cdot \bar{s}_j}{m_i m_j r_{ij}^3} \quad (90)$$

The effect is so large that it must be p-adically first order and the generalization of the mass splitting formula is rather obvious:

$$\Delta s = \sum_{pairs} S(i, j) \bar{s}_i \cdot \bar{s}_j \quad (91)$$

The coefficients  $S_{ij}$  depend must be such that integer valued  $\Delta s$  results and Planck masses are avoided: this makes the model highly predictive. Coefficients can depend both on quark pair and on hadron since the size of hadron need not be constant. In any case, very limited range of possibilities remains for the coefficients.

In principle also interaction between electroweak isospins is possible in hadron. The understanding of the mass of  $\Omega$  seems to be difficult unless this interaction is present. This interaction seems to occur mainly between electroweak isospins of fermions of same generation so that the corresponding symmetry group is tensor power of electroweak  $su(2)$  groups rather than

some larger group as in GUTs. This implies a deviation from age old  $su(3)$  picture of light hadrons.

Some examples are useful in order to make new picture familiar. For instance for  $uds$  baryon  $ud$  pair belongs to irreducible electroweak multiplet with  $J = 1$  or  $J = 0$  and  $s$  is doublet. For  $ssd$  type baryon  $ss$  pair is in  $J = 1$  or  $J = 0$  multiplet. For  $sss$  state (the troublesome  $\Omega$ ) isospins form state in  $I = 3/2$  multiplet. The remaining states ( $ccc, ccs, css$ ) of this multiplet contain charmed quarks.  $ssu$  and  $ssd$  ( $\xi$ ) baryons belong to  $I = 0$  strange multiplet.

The general form of the electroweak isospin-isospin interaction is same as of color magnetic interaction. Each generation pair  $(g_1, g_2)$ ,  $g_i = 0, 1, 2$  can have different interaction strength  $K(g(i), g(j))$ .

$$\Delta s = \sum_{pairs} K(g(i), g(j)) \bar{I}_i \cdot \bar{I}_j \quad (92)$$

and no Planck mass requirement gives constraints on the values of the matrix  $K$ .

#### 4.4.1 Baryonic case

Consider first the determination of  $S(i, j)$  and  $K(g(i), g(j))$  in case of baryons. The general splitting pattern for baryons resulting from color Coulombic, spin-spin and isospin-isospin interactions is given by the following table. It should be noticed that  $\Sigma$  baryons are not eigenstates of total electroweak isospin but superpositions of  $I = 3/2$  and  $I = 1/2$  states with equal weights. Furthermore,  $\Omega$  is  $I = 3/2$  state and the isospin of  $ss$  pair in  $\Xi$  is  $I_{12} = 1$ .

baryon	J	$J_{12}$	I	$I_{12}$	$\Delta s^I$	$\Delta s^{spin}$	$s_{eff}$
N	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{3}{4}K(0,0)$	$-\frac{3}{4}S(d,d)$	18
$\Delta$	$\frac{3}{2}$	1	$\frac{3}{2}$	1	$\frac{3}{4}K(0,0)$	$\frac{3}{4}S(d,d)$	31
$\Lambda = uds$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{3}{4}K(0,0)$	$-\frac{3}{4}S(d,d)$	25
$\Sigma = uds$	$\frac{1}{2}$	0	$\frac{1}{2} \oplus \frac{3}{2}$	1	$\frac{1}{4}K(0,0) - \frac{1}{4}K(0,1)$	$-\frac{3}{4}S(d,d)$	29
$\Sigma^*$	$\frac{1}{2}$	0	$\frac{1}{2} \oplus \frac{3}{2}$	1	$\frac{1}{4}K(0,0) - \frac{1}{4}K(0,1)$	$\frac{1}{4}S(d,d) + \frac{1}{2}S(d,s)$	40
$\Xi = ssd$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{4}K(1,1) - K(0,1)$	$-\frac{3}{4}S(s,s)$	36
$\Xi^*$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$-\frac{3}{4}K(1,1)$	$\frac{1}{4}S(s,s) + \frac{1}{2}S(d,s)$	48
$\Omega$	$\frac{3}{2}$	1	$\frac{3}{2}$	1	$\frac{3}{4}K(1,1)$	$\frac{3}{4}S(s,s)$	58

Table 4.8. Color Coulombic, spin-spin and isospin-isospin splittings for baryons.

Spin-spin and isospin-isopin splittings are deduced from the formulas

$$\begin{aligned}
\Delta s^{spin} &= S(q(1), q(2)) \left( \frac{J_{12}(J_{12} + 1)}{2} - \frac{3}{4} \right) \\
&+ \frac{1}{4} (S(q(1), q(3)) + S(q(2), q(3))) (J(J + 1) - J_{12}(J_{12} + 1) - \frac{3}{4}) \\
\Delta s^I &= K(g(1), g(2)) \left( \frac{I_{12}(I_{12} + 1)}{2} - \frac{3}{4} \right) \\
&+ \frac{1}{4} (K(g(1), g(3)) + K(g(2), g(3))) (I(I + 1) - I_{12}(I_{12} + 1) - \frac{3}{4})
\end{aligned} \tag{93}$$

where  $J_{12}$  is the angular momentum eigenvalue of the 'first two quarks', whose value is fixed by the requirement that magnetic moments are of correct sign. All baryons are eigenstates of  $I$  and  $I_{12}$  except  $\Sigma = uds$ , where  $ud$  is  $I_{12} = 1$  state so that state is superposition of  $I = 1/2$  and  $I = 3/2$  states with equal weights.

The masses determine the values of the parameters uniquely if one assumes that color binding energy is constant:  $c(q(i), q(j)) = c$  Assuming that the values of  $s(ef\bar{f}, q(i))$  are  $s(ef\bar{f}, d) = s(ef\bar{f}, u) = 9$  and  $s(ef\bar{f}, s) = 16$  one obtains the following table



$K(0, 0)$	$K(0, 1)$	$K(1, 1)$	$S(d, d)$	$S(d, s)$	$S(s, s)$	$c$
4	0	10	$5 - \frac{1}{3}$	$12 + \frac{2}{3}$	$7 - \frac{1}{3}$	$-1 + \frac{1}{6}$
4	0	10	4	12	6	-1

Table 4.9. The values of the parameters characterizing baryonic spin-spin and isospin splittings of baryons in order  $O(p)$ . The lower row gives the best integer values for the parameters.

These parameter values in the table produce the first order contributions exactly. If one requires that the parameters are integers lead to errors in mass prediction below two per cent. Remarkably, the values of  $K(i, j)$  are integers.  $K(0, 1)$  vanishes as in bosonic case, where the vanishing is implied by the smallness of CP breaking and this suggests than in first order the isospin-isospin interaction between different generations vanishes: a possible explanation is that exchange force is in question.

baryon	N	$\Delta$	$\Lambda$	$\Sigma$	$\Sigma^*$	$\Xi$	$\Xi^*$	$\Omega$
$s_{eff}$	18	30	25	29	40	36	48	58
$s_{eff}(exp)$	18	31	25	29	39	36	47	57

Table 4.10. Predictions for  $s(eff, B)$  assuming integer values for spin-spin and isospin-isospin interaction parameters. The errors induced in the prediction of masses are below two per cent.

#### 4.4.2 Mesonic case

For mesons the general splitting pattern is given by the following table.

meson	$\Delta s^I$	$\Delta s^{spin}$	s
$\pi$	$\frac{1}{4}K(0, M)$	$-\frac{3}{4}S(d, d, M)$	0
$\rho$	$\frac{1}{4}K(0, M)$	$\frac{1}{4}S(d, d, M)$	12
$\eta$	$-\frac{3}{4}K(0, M)$	$-\frac{3}{4}S(d, d, M)$	6
$\omega$	$-\frac{3}{4}K(0, M)$	$\frac{1}{4}S(d, d, M)$	12
$K^\pm, K^0(CP = 1)$	$\frac{1}{4}K(0, 1, M)$	$-\frac{3}{4}S(d, s, M)$	5
$K^0(CP = -1)$	$-\frac{3}{4}K(0, 1, M)$	$-\frac{3}{4}S(d, s, M)$	5
$K^{*,\pm}, K^{*,0}(CP = 1)$	$\frac{1}{4}K(0, 1, M)$	$\frac{1}{4}S(d, s, M)$	16
$K^{*,0}(CP = -1)$	$-\frac{3}{4}K(0, 1, M)$	$\frac{1}{4}S(d, s, M)$	16
$\eta'$	$-\frac{3}{4}K(1, M)$	$-\frac{3}{4}S(s, s, M)$	19
$\Phi$	$-\frac{3}{4}K(1, M)$	$\frac{1}{4}S(s, s, M)$	21
$\eta_c$	$-\frac{3}{4}K(1, M)$	$-\frac{3}{4}S(c, c, M)$	184
$\Psi$	$-\frac{3}{4}K(1, M)$	$\frac{1}{4}S(c, c, M)$	200
$D^\pm, D^0(CP = 1)$	$\frac{1}{4}K(0, 1, M)$	$-\frac{3}{4}S(d, c, M)$	72
$D^0(CP = -1)$	$-\frac{3}{4}K(0, 1, M)$	$-\frac{3}{4}S(d, c, M)$	72
$D^{*,\pm}, D^{*0}(CP = 1)$	$\frac{1}{4}K(0, 1, M)$	$\frac{1}{4}S(d, c, M)$	83
$D^{*0}(CP = -1)$	$-\frac{3}{4}K(0, 1, M)$	$\frac{1}{4}S(d, c, M)$	83

Table 4.11. Isospin and spin-spin splitting pattern for mesons.

The values of  $S(M, i, j)$  and  $K(M, i, j)$  can in principle deduced from the observed mass splittings. Complications however result from the possible mixing of  $(I = 0, J = 0)$  and  $(I = 0, J = 1)$  mesons.

- From  $\rho - \pi$  mass splitting one has  $S(M, d, d) = 12$ .
- If  $\omega$  suffers no mixing with  $\Phi, \Psi, Ypsilon..$  the equality  $s(\omega) = s(\rho)$  implies  $K(M, 0, 0) = 0$  and  $\Delta s_c(M) = -9$ .
- $K^* - K$ -splitting gives  $S(M, d, s) = 16 - 5 = 11$  not far from  $S(M, d, s) = 12$ .
- $D^* - D$  mass difference  $s(D^*) - s(D) = 83 - 72 = 11$  gives prediction  $S(M, d, c) = 12$ . Note that again there is discrepancy of one unit in the prediction of mass splitting.
- Smallness of CP breaking in  $K - \bar{K}$  system implies  $K(0, 1) = 0$ . Same applies to  $D - \bar{D}$  and  $B - \bar{B}$  so that all nondiagonal elements of  $K(i, j)$  must vanish, which in turn suggests that isospin-isospin interaction for mesons vanishes identically in first order.
- $F - D$  mass difference gives  $S(M, s, c) = \frac{4}{3}(s(s) - s(d) - 1)$  and gives

$S(M, s, c) = 16$  for  $s(s) = 14$  with predicted splitting too large by one unit and  $S(s, c) = 20$  for  $s(s) = 16$ .

There are some discrepancies in the simplest picture without mixing effects.

- a)  $K - \pi$  mass splitting assuming  $s(eff, s) = 16$  is  $s(K) - s(\pi) = 7 > 5$ .
- b)  $\omega - \eta$  splitting  $s(\omega) - s(\eta) = 6$  gives, assuming that  $\eta$  does not contain  $s\bar{s}$  pair,  $s(M, d, d) = 8$ , which is smaller than the value suggested by  $\rho - \pi$  splitting.  $\Phi - \eta'$  mass difference  $s(\Phi) - s(\eta') = 21 - 19 = 2$  suggests  $S(M, s, s) = 4$  or  $s(M, s, s) = 0$ . Both values are suspiciously small.

The mixing of  $\eta$  and  $\eta'$  and  $\eta_c$  provides a nice explanation of the discrepancies.

- c) The mass of *Ypsilon* meson for  $s(eff, s) = 16$  is predicted to be  $s(eff, Ypsilon) = 2s(b) - 9 + \frac{1}{4}(S(b, b) + K(2, 2)) = 1868$ , which gives  $S(b, b) + K(2, 2) = 4 \cdot 139 = 556$ , which is suspiciously large as compared with the values associated with lighter quarks.

One can consider two manners to get rid of the  $K - \pi$  discrepancy.

- a) The first possibility is the mixing of  $\omega$  with its higher mass companions, which allows  $K(M, 0, 0) = 8$  and correct  $K - \pi$  mass difference. This however leads to negative mass squared for  $\eta$ :  $s(\eta) = -8$  in absence of mixings and it therefore seems that  $K(M, 0, 0) \leq 0$  is the only reasonable possibility.  $K(M, 0, 0) < 0$  in turn implies  $s(eff, \omega) > s(eff, \rho)$  and mixing effects can make the situation only worse so that  $K(0, 0) = 0$  is the only possibility and the mixing of  $J = 1$  mesons does not help. Of course,  $\Psi$  and *Ypsilon*  $t\bar{t}$  ( $J = 1, I = 0$ ) mesons could mix so that one could avoid anomalously large values of  $s(M, b, b)$ .
- b) The parameter  $n_2(eff, d)$  fixing the mass of strange quark has negligibly small effect on the values of CKM matrix elements and therefore one could give up the assumption that topological mixing matrices are identical for mesons and baryons and assume that for mesonic quark one has

$$s(eff, s, M) = 14 \tag{94}$$

instead of  $s(eff, s, B) = 16$ . This implies correct  $K - \pi$  mass difference. This increases the value of  $s(eff, b, M)$  by 32 units but the effect is not large enough and one must still have  $S(b, b) + = 4 \cdot 75 = 300$  in order to get *Ypsilon* mass correctly.

If one assumes that  $K(M, i, j) = 0$  identically and no mixing for  $J = 1$  states one obtains correct masses for  $J = 1$  mesons  $\Phi, \Psi, B$  using following values of parameters

$$\begin{aligned} S(M, s, s) &= 8 \\ S(M, c, c) &= 40 \\ S(M, b, b) &= 300 \end{aligned} \tag{95}$$

The predictions for the masses of  $J = 0$  partners are systematically too small

$$\begin{aligned} s(eff, \eta) &= 0 < 6 \\ s(eff, \eta') &= 13 < 19 \\ s(eff, \eta_c) &= 159 < 185 \\ s(eff, \eta_b) &= 1568 \end{aligned} \tag{96}$$

To get rid of the remaining discrepancies one must assume mixing between neutral pseudo scalar mesons  $\eta, \eta', \eta_c, \eta_b, \eta_t$ . If mass squared is simply the quantum mechanical expectation for mass squared of  $\eta$  and  $\eta'$  and  $\eta_c$  implies that the masses of  $\eta$  and  $\eta'$  are actually smaller than the masses of their mixed partners and the effective value of  $s(M, i, j)$  derived from mass difference is expectation value, which contains also the contribution of quark mass differences. In the simplest scenario mesonic isospin-isospin interaction vanishes in first order for the lowest two generations at least,  $\eta$  mixes with  $\eta'$  (pure  $s\bar{s}$  pair) only and  $\eta'$  mixes with  $\eta$  and  $\eta_c$  only,....

$$\begin{aligned} \eta_{phys} &= c_1\eta + s_1\eta' \\ \eta'_{phys} &= c_2(-s_1\eta + c_1\eta') + s_2\eta_c \\ \eta^c_{phys} &= c_3(-s_2(-s_1\eta + c_1\eta') + c_2\eta_c) + s_3\eta_b \\ &\dots \end{aligned} \tag{97}$$

Various masses come out correctly if one has

$$s_1^2 = \frac{s(\eta_{phys})}{s(\eta')} = \frac{6}{13} \simeq 0.462$$

$$\begin{aligned}
s_2^2 &= \frac{s(\eta'_{phys}) - c_1^2 s(\eta')}{s(\eta_c) - c_1^2 s(\eta')} = \frac{12}{152} \simeq 0.079 \\
s_3^2 &= \frac{s(\eta_{phys}^c) - c_2^2 s(\eta^c) - c_1^2 s_2^2 s(\eta')}{s(\eta_b) - s(\eta_c) c_2^2 - c_1^2 s_2^2 s(\eta')} \\
&= \frac{152 \cdot 185 - 140 \cdot 159 - 7 \cdot 12}{152 \cdot 1568 - 1559 \cdot 140 - 7 \cdot 12} = \frac{5776}{216426} \simeq .0267 \\
&\dots
\end{aligned} \tag{98}$$

To sum up, the discussion leads to a definite predictions for  $\eta, \eta', \eta_c$  mixing and fixes the values of the parameters  $S(i, j)$  and  $K(i, j)$

S(d,d)	S(d,s)	S(d,c)	S(s,s)	S(s,c)	S(c,c)	S(b,b)
12	12	12	8	16	40	300

Table 4.12. The values of spin-spin interaction parameters for mesons. Isospin-isospin interaction is assumed to vanish in first order for mesons.

The first order contributions to the masses of 'diagonal' mesons are reproduced correctly. Situation is different for 'non-diagonal' mesons.  $K^* - K$  and  $D^* - D$  mass splittings are predicted to be too large by one unit and there are large errors in the predictions for  $D, F$  and  $B$  meson masses, which will be discussed separately.

meson	$\pi$	$\rho$	$\eta$	$\omega$	$K$	$K^*$	$\eta'$	$\Phi$	$\eta_c$	$\Psi$	Ypsilon
$s(eff, M, exp)$	0	12	6	12	5	16	19	21	185	200	1868
$s(eff, M, pred)$	0	12	6	12	5	17	19	21	185	200	1868

Table 4.13. Predictions for  $s(eff)$  for mesons assuming integer values for spin-spin and isospin-isospin interaction parameters.  $D, F$  and  $B$  mesons are not included in the table.

The values of parameters allow to deduce the parameter  $\Delta s_c$  characterizing the strength of color Coulombic interaction in order  $O(p)$ . The results are

$$\begin{aligned}
\Delta s_c(B) &= 3c(B) = -3 + \frac{1}{2} \\
\Delta s_c(M) &= c(M) = -9
\end{aligned} \tag{99}$$

These parameters fix the the masses of proton and pion and it would be nice if one could really predict the values of these parameters. A QCD inspired estimate for the color binding energies

$$\begin{aligned}\Delta m_c(B) &= -\frac{3C}{2} \frac{\alpha_s(r_B)}{r_B} \\ \Delta m_c(M) &= -\frac{C}{2} \frac{\alpha_s(r_M)}{r_M}\end{aligned}\tag{100}$$

where  $C(3) = 8/3$  is the value of Casimir operator for triplet representation, is consistent with TGD estimate provided the value of color coupling strenght in mesonic length scale  $r_M$  larger than baryonic length scale is much larger than in baryonic length scale  $r_B$ . For proton the estimate  $3/18 = \Delta m^2/m^2 = 2\Delta m_c/m$  gives estimate  $\alpha_s \sim r_B m_p/16 \sim 0.06$ , which is reasonably near to  $\alpha_s \sim 0.12$ . For pion the corresponding estimate  $\alpha_s \sim 3m_p r_M/(4\sqrt{2})$ , which is of order one.

## 4.5 Condensate level mixing

The following table shows the predictions for the parameter  $s$  determining mass squared in order  $O(p)$  for heavy hadrons containing charmed and bottom quarks. Topological mixing and color magnetic hyperfine and isospin-isospin splittings (which are rather small effects) are taken into account, when corresponding parameters are known.

hadron	$s$	$s_{exp}$
$\Lambda_c$	$s(\lambda) + s(c) - s(s) = 108$	108
$\Lambda_b$	$s(\lambda) + s(b) - s(s) = 878$	614
$D$	$s(c) - \frac{3}{4}S(d, c) = 90$	72
$B$	$s(b) - \frac{3}{4}S(d, b) = 869 - 32\Delta - \frac{3}{4}S(d, b)$	588

Table 4.14. The predictions for certain hadron masses compared with experimental masses for  $s(s) = 16 - \Delta$  ( $\Delta = 2$  is suggested by  $K - \pi$  mass difference).

From the table it is clear the values of mass squared of hadrons ( $B$  and  $\Lambda_b$ ) containing one bottom quark are much lower than the predictions following

from the proposed scenario. Hyper fine splittings and isospin-isospin interactions certainly cannot help in the problem. For  $D$  meson containing one  $c$  quark (hyper fine splitting has been taken into account), the effect seems to be present but is considerably smaller. For  $\Lambda_c$  the first order prediction is exact.

A possible explanation is that  $b$  quark spends part of its time on  $k = 107$  level rather than  $k = 103$  level so that average mass squared is reduced. Since the prediction for the mass of Ypsilon is essentially correct it seems that inside Ypsilon the mechanism is not at work. The only manner to escape contradiction is to assume that  $b$  quark condenses on  $u, d$  or  $s$  quark, whose primary condensate level is also  $k = 107$ , which in turn condenses on hadronic level, which corresponds to  $p$  near  $M_{107}$  and has also  $k = 107$ . Inside Ypsilon there is no place for  $b$  to condense and therefore Ypsilon remains heavy. The same mechanism applies to  $\Psi$ . If quark prefers to condense on antiquark rather than quark then one could understand why the effect is not seen for  $\lambda_c$ . One must however remember that the effect is small for  $c$  quark and some other explanation might work equally well.

Condensate level mixing can be described quantum mechanically in same manner as the topological mixing. Physical state at  $k = 107$  is superposition of states, where the primary condensation level of  $b$  ( $c$ ) quark is either  $k = 103$  or  $k = 107$  and mass squared is quantum mechanical expectation value for masses for these two primary condensation levels

$$\begin{aligned}
|b_{phys}\rangle &= \cos(\phi)|b_{103}\rangle + \sin(\phi)|b_{107}\rangle \\
s_{phys}(b) &= \cos^2(\phi)s(b, k = 103) + \sin^2(\phi)s(b, k = 107) \\
s(b, 103) &= 5 + 2^4(54 + \Delta) \\
s(b, 107) &= 5 + 54 + \Delta \\
s(s) &= 16 - \Delta
\end{aligned} \tag{101}$$

This equation has obvious generalization to the case of  $c$  quark with  $s(c) = 3 + 6 \cdot 2^4$  and one obtains the following mass formula

$$\begin{aligned}
s_{phys}(b) &= 59 + \Delta 15 \cdot (54 + \Delta) \cos^2(\phi_b) \\
s_{phys}(c) &= 9 + 15 \cdot 6 \cos^2(\phi_c)
\end{aligned}$$

$$\begin{aligned}
\cos^2(\phi_b) &= \frac{n_b}{15 \cdot (54 + \Delta)} \\
\cos^2(\phi_c) &= \frac{n_c}{90}
\end{aligned} \tag{102}$$

No Planck mass requirement implies the expressions for mixing angle.

The requirement that  $B$  and  $D$  masses are predicted correctly implies

$$\begin{aligned}
n_b &= 529 + \Delta + \frac{3}{4}S(M, d, b) \\
n_c &= 72 \\
\sin^2(\phi_b) &= \frac{281 + 14\Delta - \frac{3}{4}S(d, b)}{810 + 15\Delta} \\
\sin^2(\phi_c) &= \frac{1}{5}
\end{aligned} \tag{103}$$

where  $\Delta s^I(D/B) = 0$  (no CP breaking in first order) and  $\Delta s^{spin}(D) = -9$  are used.

In the baryonic case one can derive constraints on the values of the mixing angles from  $\Lambda_b$  and  $\Lambda_c$  masses. For c quark baryonic mixing angle vanishes. A possible explanation is that condensation of c quark to anti quark is more probable process than condensation on quark. The requirement that baryonic mixing angle for  $b$  quark is identical with the mesonic one fixes the value of the mesonic spin-spin interaction parameter  $S(M, d, b)$ , the value of the mixing angle and the value of  $s(eff, mix, b)$

$$\begin{aligned}
S(M, d, b) &= \frac{4}{3}(s(\Lambda_b) - s(B) + s(s) - s(\Lambda)) \rightarrow 32 \\
\sin^2(\phi_b) &= \frac{57}{168} \simeq 0.34 \\
s(eff, mix, b) &= s(B) + \frac{3}{4}S(M, d, b) = 612
\end{aligned} \tag{104}$$

The result means that  $b$  quark spends about one third of its time on lower condensate level.



## 4.6 Summary: hadronic mass formula in first order

For the convenience of the reader the hadronic mass formula in first order is summarized. The value of the coefficient  $s(H)$  in  $M^2(H) = s(H)p + \dots$  giving leading contribution to the hadron mass (pion being exception) is

$$s(H) = \sum_i s(q_i, eff) + \sum_{pairs} S(i, j) \bar{s}_i \cdot \bar{s}_j + \sum_{pairs} K(g(i), g(j)) \bar{I}_i \cdot \bar{I}_j + (195)$$

a) The values of  $s(q_i, eff)$  correspond to quark masses, when topological mixing effects are taken into account. For hadrons containing both charmed or b quarks and u,d,s quarks there is also the mixing of primary condensation levels present and different value of  $s(q_i, eff)$  must be used.  $K - \pi$  mass difference is too large by 2 units unless one uses  $s(M, s) = 14$  instead of the baryonic value  $s(B, s) = 16$ : CKM matrices are essentially identical in both cases.

baryonic quark	d	s	b	u	c	t
$s(eff)$	9	16	869	9	99	$3 + 57 \cdot 2^{10}$
$s(eff, mix)$	9	16	612	9	99	?
mesonic quark	d	s	b	u	c	t
$s(eff)$	9	14	901	9	99	$3 + 57 \cdot 2^{10}$
$s(eff, mix)$	9	14	612	9	81	?

Table 4.15. The known values of  $s(eff, q_i)$  in the scenario providing a solution for the spin crisis of proton. The second row contains the effect of condensate level mixing in hadrons containing  $u, d$  or  $s$  quark and  $c$  quark or  $b$  quark. For baryons the effect is not present for  $c$  quark and the value given in parenthesis must be used. For mesons  $s(s) = 16$  would predict  $K - \pi$  mass squared difference two units too large. For top quark  $k = 97$  condensation level is assumed in the mass formula.

b) The elements of the integer matrices  $S(i, j)$  and  $K(g(i), g(j))$  parametrize color magnetic spin-ispin interaction and isospin-isospin interaction and are given in the tables below.

$K(0,0)$	$K(0,1)$	$K(1,1)$	$S(d,d)$	$S(d,s)$	$S(s,s)$	$\Delta s_c$
4	0	10	$5 - \frac{1}{3}$	$12 + \frac{2}{3}$	$7 - \frac{1}{3}$	$-1 + \frac{1}{6}$
4	0	10	4	12	6	-1

Table 4.16. Estimates for the parameters describing spin-spin and isospin-isospin interactions for baryons. Fractional valued parameters reproduce low lying hadron masses exactly and integer valued parameters reproduce masses with errors not larger than  $|\Delta s| = 1$ .

S(d,d)	S(d,s)	S(d,c)	S(s,s)	S(s,c)	S(c,c)	S(d,b)	S(b,b)
12	12	12	8	16	40	32	300

Table 4.17. Estimates for the parameters describing spin-spin interactions for mesons assuming  $s(eff, ss) = 14$  for mesonic strange quark. Isospin-isospin interaction vanishes for mesons in lowest order.

c) The values of the integer  $\Delta s_c$  parametrizing color Coulombic binding energy for baryons and mesons are

$$\begin{aligned}
\Delta s_c(B) &= -3 + \frac{1}{2} \\
\Delta s_c(M) &= -9
\end{aligned} \tag{106}$$

If the parameters are assumed to be integers one has  $\Delta s_c(B) = -3$ .

d) For neutral pseudoscalar mesons  $\eta, \eta', \eta_c$  also mixing effects are important in first order and the values are obtained by calculating the masses of  $\eta, \eta', \eta_c$  using previous formulas and using following formulas to take mixing effects into account

$$\begin{aligned}
s(\eta_{phys}) &= s_1^2 s(\eta') \\
s(\eta'_{phys}) &= c_1^2 c_2^2 s(\eta') + s_2^2 s(\eta_c) \\
s(\eta_{phys}^c) &= c_1^2 s_2^2 c_3^2 s(\eta') + c_1^2 c_2^2 c_3^2 s(\eta_c) + s_3^2 s(\eta_b) \\
s_1^2 &= \frac{6}{13} \\
s_2^2 &= \frac{12}{52}
\end{aligned}$$

$$s_3^2 = \frac{5776}{216426} \quad (107)$$

In baryonic case first order contributions are reproduced exactly for fractional values of  $S(i, j)$ ,  $K(i, j)$  and for integer values errors are not larger than  $|\Delta s| = 1$ . If  $s(s, M) = 14$  is assumed the errors in mesonic sector are associated with spin-spin splittings of nondiagonal mesons  $K - K^*$ ,  $D - D^*$ , which are predicted to be too large by one unit. For  $s(s, M) = 16$  the value of  $s(K)$  is predicted to be two units too large. One cannot exclude the possibility of some unidentified contribution to the masses of nondiagonal mesons.

#### 4.7 Second order contribution to mass squared and isospin splittings

Second order terms in cm contribution to the masses of the quarks imply definite isospin splitting pattern for hadron masses. The splittings are not in accordance with experimental splittings as the last columns of the table below show. Proton-neutron mass squared difference is of correct sign but too large by a factor 16. For  $DDD$  ( $\Delta^-$ ) configuration  $X$  vanishes whereas experimentally the mass in general increases with decreasing charge. There are several sources to second order term in mass squared.

a) Coulombic splitting: the general form of Coulombic mass splitting in second order is given by

$$\Delta X(Coul) = \sum_{pairs} Q_k Q_l D_e(k, l) \quad (108)$$

b) Electromagnetic spin-spin interaction. The general form of the magnetic interaction is

$$\Delta X(magn) = \sum_{pairs} Q_k Q_l \bar{S}_k \cdot \bar{S}_l D_m(k, l) \quad (109)$$

The strongest assumption is that the parameters  $D_e(k, l)$  and  $D_m(k, l)$  are same for all quark pairs and all baryons and mesons respectively. A more realistic assumption is that  $D_e$  are different for mesons and baryons. Still one

step towards realism is to notice that Coulombic and magnetic energies are inversely proportional to the first and third power of electromagnetic radius of hadron respectively so that an appropriate hadron dependent scaling factor could be present in the parameter.  $D_m(k, l)$  could also depend on quark pair (being inversely proportional to the product of quark masses in quark model). An interesting possibility is that  $D_m(k, l)$  is proportional to the matrix  $S(k, l)$  associated with color hyper fine interaction. This could be the case since the matrix in question summarizes information about geometry (average quark-quark distance) and quark masses. Therefore an interesting hypothesis to be tested is

$$D_m(k, l) = dS(k, l) \quad (110)$$

There is considerable information on elements of  $S(k, l)$  at use. In the following it is useful to take the notational conventions

$$\begin{aligned} C_m(k, l) &= \frac{D_m(k, l)}{36} \\ C_e(k, l) &= \frac{D_e(k, l)}{9} \end{aligned} \quad (111)$$

at use.

There are also interactions, which produce constant shifts depending on isospin and spin multiplet, which are important in p-adic regime since the addition of small power of two to second order term may change the real counterpart of term radically.

a) Second order contribution to color spin-spin interaction. This interaction introduces only constant shift between  $J = 3/2$  and  $J = 1/2$  baryons and between  $J = 1$  and  $J = 0$  mesons and the form of this contribution is same as in  $O(p)$  case.

b) Color Coulombic interaction. This introduces only constant shift for the multiplets. Similar shift results from the sea contribution to mass squared, which is automatically of order  $O(p^2)$ .

c) Isospin-isospin interaction. If the interaction is present for the fermions of same generation the interaction produces characteristic splittings between different isospin multiplets.

At this stage the best manner to treat these terms is to allow a constant shift depending on electroweak and spin multiplet.

The assumption that isospin splittings result from  $O(p^2)$  effects has important consequence: the value of  $s(H)$  is same for all hadrons inside isospin multiplet. This requirement allows one to derive stringent bounds on the value of the fundamental mass scale  $m_0^2$ . Electron mass fixes in principle the value of  $m_0^2$  but second order corrections to electron mass (topological mixing, Coulombic energy) induce changes in mass scale and for heavier hadrons small change can induce even a change in the value of  $s$  and the values of  $X$  suffer shift (change is proportional to  $\delta M^2$ , where  $\delta$  is the change in basic mass scale). A possible manner to get rid of inaccuracy is to deduce the scale factor by comparing two known hadrons known to have identical second order correction to mass squared (hadrons of  $M_{89}$  hadron physics are ideal in this respect!). The fit of masses based on electron mass formula without Coulombic and mixing corrections has some flaws: for instance, the values of  $s$  differ by one unit for  $\Xi^-$  and  $\Xi^0$ : this is a clear indication about the presence of corrections to electron mass. In order to get consistency one must increase or decrease basic mass scale a little bit. Since corrections to electron mass seems to be positive the decrease of mass scale comes in question so that  $s$  and  $X$  for heavier baryons tend to increase. The minimal change in scale corresponds to

$$\begin{aligned} m_0^2 &\rightarrow m_0^2 \frac{5 + 2/3}{5 + 2/3 + \delta} \\ \delta &\simeq 0.0515 \end{aligned} \tag{112}$$

An attractive possibility is that  $\delta$  super position of a small number of powers of two: simplest alternative is  $\delta = 1/64 \simeq 0.0516$ . With this assumption  $\Delta^-$  has still  $s$  larger by one unit than its companions and has  $X(\Delta^-) = 1/64$ : the experimental uncertainty in  $\Delta^- - \Delta^{++}$  splitting is much larger than the reduction in  $\Delta^-$  mass needed to get rid of the difficulty. The proposed value of  $\delta$  leads to an essentially correct prediction for  $Z^0$  mass and  $W$  mass is predicted correctly taking Coulombic corrections into account.

The reliable determination of the parameters  $C_e, C_m$  and shifts  $\Delta(B)$  depending on multiplet is not a trivial task and there are some delicate points involved.

- a) The real counterpart of the second order term is very sensitive to the changes in its p-adic counterpart: relatively small fractional contribution, say constant shift, could change totally the splitting scenario unless it corresponds to sufficiently large negative power of 2. The reason is that the p-adic power expansion of fractional numbers has as its lowest term large p-adic number of order  $p$ . Two large terms of this kind just below  $p = M^{107}$  can hower sum up to number which is essentially zero modulo  $p$ .
- b) The appearance of  $1/60$  factors in the cm contribution to second order term suggests that natural unit for second order term is  $1/60$ . This is not the case. The point is that  $1/60$  is not expressible as sum of finite number of powers of two and this in turn implies the existence of several values of  $X_{eff}$  with the property that  $pX_{eff}$  maps to same  $X_{eff}$  in canonical correspondence. For instance, for  $n = 8$  p-adic numbers  $8p/120, 36p/120$  and  $68p/120$  map to  $n/60$  for  $p = M_{107}$ . A natural unit for second order term is  $1/64$  since it corresponds to the natural unit of mass squared for the representations of Super Virasoro with broken conformal symmetry. The second nice feature is that the p-adic counterpart of  $X/64$  for  $X < 64$  is just  $X/64$  since the inverse of 64 is  $2^{101}$  in good approximation.
- c) The calculation of first order terms for p-adic expansions of fractional number  $m/n$  reduces to that of finding the inverse of the denominator  $n$  in the finite field  $G(p = M_{107}, 1)$  consisting of integers smaller than  $p$ . The inverses of following numbers will be needed often:

$$\begin{aligned}
\frac{1}{3} &= 2^{106} + 2^{104} + \dots \\
\frac{1}{5} &= 2^{105} - 2^{103} + \dots \\
\frac{1}{25} &= 2^{103} - 2^{102} + 2^{100} - 2^{99} - 2^{98} + \dots \\
\frac{1}{15} &= 2^{103} + 2^{99} + \dots \\
\frac{1}{120} &= 2^{100} + 2^{96} + \dots
\end{aligned} \tag{113}$$

Using these one can easily generate the real counterparts of the second order term in mass squared.

#### 4.7.1 Baryonic case

Consider first the determination of the parameters  $C_m(d, d) = C_m$ ,  $C_e(d, d) = C_e$  and  $\Delta X_0$  for nucleons and  $\Delta$  resonances. The following table summarizes various electromagnetic contributions to the coefficient of second order term for nonstrange baryons. The coefficient of various contributions are normalized to integers in order to facilitate calculations: one has  $E_m \equiv 3C_m$  and  $C_e = D_e/9$ . Listed are also the p-adic counterparts of  $X_{eff}$  in order to facilitate calculations.

baryon	$\frac{\Delta X(magn)}{E_m}$	$\frac{\Delta X(coul)}{C_e}$	X	$64X_{eff}$
p	2	0	$\frac{13}{30}$	25
n	2	-1	$\frac{18}{30}$	28
$\Delta^{++}$	4	4	$\frac{24}{30}$	41
$\Delta^+$	0	0	$\frac{13}{30}$	54
$\Delta^0$	-1	-1	$\frac{18}{30}$	50
$\Delta^-$	1	1	0	$\leq 63, \geq 41$

Table 4.18. Various contributions to iso-spin splitting for nonstrange baryons.

The masses are taken from the most recent Particle Data Tables and the values of  $X_{eff}$  are subject to a shift of  $\Delta X = \pm 2$  resulting from the error in average mass. The relative error for  $\Delta^- - \Delta^{++}$  mass difference in older Particle Data Tables and is almost as large as the mass difference. In the most recent Particle Data Table this mass difference is not reported. The use of the rather large average value of  $\Delta^- - \Delta^{++}$  mass difference would give  $s(\Delta^-) = s(\Delta^{++}) + 1$ , which cannot hold true if isospin splittings are second order effect. Fortunately, only a slight change in  $\Delta^-$  mass saves the situation.

a) Proton and neutron masses give the conditions

$$\begin{aligned}
C_e &= X_{eff}(p) - X_{eff}(n) + \frac{5}{30} = -\frac{3}{64} + \frac{5}{30} \\
\Delta(N) &= -2E_m + X_{eff}(p) - \frac{13}{30} = -2E_m + \frac{25}{64} - \frac{13}{30}
\end{aligned} \tag{114}$$

b) One can solve the values of the parameters  $E_m, \Delta(\Delta)$  from  $\Delta$  masses and obtains two  $\Delta$  masses as predictions. It is useful to express the solution in terms of  $\Delta^0 - \Delta^+$  mass difference  $\delta$ .

$$\begin{aligned}
E_m &= \frac{3}{64} + \delta = \frac{7}{64} \\
\Delta(\Delta) &= X(\Delta^+) - \frac{13}{30} \\
X(\Delta^{++}) &= X(\Delta^+) + \frac{1}{30} - 4\delta = \frac{41}{64} + \frac{1}{30} \\
X(\Delta^-) &= X(\Delta^+) - \frac{2}{15} + \delta = \frac{56}{64} - \frac{2}{15} \\
\delta &= X(\Delta^+) - X(\Delta^0) = \frac{4}{64}
\end{aligned} \tag{115}$$

From the formulas for the masses it is clear that masses are very sensitive to the value of the mass difference  $\delta$ .

d) The masses of  $\Delta^{++}$  and  $\Delta^-$  come out as follows:

baryon	X	$(pX_{tot})_R$	$X_{eff}$
$\Delta^{++}$	$\frac{38}{64} + \frac{1}{30}$	$\frac{40}{64} + \frac{1}{15 \cdot 32}$	$\frac{41}{64}$
$\Delta^+$	$\frac{54}{64}$	$\frac{54}{64}$	$\frac{54}{64}$
$\Delta^0$	$\frac{50}{64}$	$\frac{50}{64}$	$\frac{50}{64}$
$\Delta^-$	$\frac{58}{64} - \frac{2}{15}$	$\frac{50}{64} - \frac{1}{120}$	?

Table 4.19. The predictions for the isospin splittings of  $\Delta$  resonances.

$\Delta^{++}$  mass is predicted correctly within experimental uncertainties. The masses of  $\Delta^-$  and  $\Delta^0$  are predicted to be identical with good accuracy: the prediction is within the error bars of the earlier Particle Data Table data on the  $\Delta^- - \Delta^{++}$  mass difference. Unfortunately Particle Data tables give no information about this mass difference.

Consider next strange baryons, for which second order contributions to mass are listed in the table below.



baryon	$\Delta X(magn)$	$\Delta X(coul)$	X	$64X_{eff}$
$\lambda$	$6C_m(d, s) + 6\Delta C_0$	$-C_e(d, s) - 2\Delta C_e^0$	$\frac{18}{30}$	63
$\Sigma^+$	$6C_m(d, s) - 12\Delta C_0$	$4\Delta C_e^0$	$\frac{13}{30}$	39
$\Sigma^0$	$6C_m(d, s) + 6\Delta C_0$	$-C_e(d, s) - 2\Delta C_e^0$	$\frac{18}{30}$	45
$\Sigma^-$	$-3C_m(d, s) - 3\Delta C_0$	$C_e(d, s) + \Delta C_e^0$	0	60
$\Sigma^{*+}$	$4\Delta C_0$	$4\Delta C_e^0$	$\frac{13}{30}$	4
$\Sigma^{*0}$	$-3C_m(d, s) - 8\Delta C_0$	$-C_e(d, s) - 2\Delta C_e^0$	$\frac{18}{30}$	4
$\Sigma^{*-}$	$3C_m(d, s) + \Delta C_0$	$C_e(d, s) + \Delta C_e^0$	0	4
$\Xi^0$	$6C_m(d, s) + 6\Delta C_1$	$-C_e(d, s) - 2\Delta C_e^1$	$\frac{18}{30}$	7
$\Xi^-$	$-3C_m(d, s) - 3\Delta C_1$	$C_e(d, s) + \Delta C_e^1$	0	29
$\Xi^{*0}$	$-3C_m(d, s) - 8\Delta C_1$	$-C_e(d, s) - 2\Delta C_e^1$	$\frac{18}{30}$	0
$\Xi^{*-}$	$3C_m(d, s) + \Delta C_1$	$C_e(d, s) + \Delta C_e^1$	0	13
$\Omega^-$	$-3C_m(s, s)$	$C_e(s, s)$	0	25

Table 4.20. Various contributions to isospin splitting for strange baryons assuming that magnetic and Coulombic interaction term is different for different quark pairs. The notations  $\Delta C_0 = C_m(d, d) - C_m(d, s)$ ,  $\Delta C_1 = C_m(s, s) - C_m(d, s)$ ,  $\Delta C_e^0 = C_e(d, d) - C_e(d, s)$  and  $\Delta C_e^1 = C_e(s, s) - C_e(d, s)$  are used.

Consider first  $\Sigma$  and  $\Sigma^*$  baryons. The isospin splittings of  $\Sigma^*$  are poorly known and one must use only  $\Sigma$ 's to deduce the values of  $C_m(d, s)$ ,  $C_e(d, s)$  and multiplet shift  $\Delta(\Sigma)$  in terms of the known parameters. Some amount of linear algebra gives

$$\begin{aligned}
\Delta(\Sigma) &= X_{eff}(\Sigma^-) + 3C_m(d, d) - C_e(d, d) = \frac{3}{32} - \frac{1}{6} \\
C_e(d, s) &= -\Delta(\Sigma) + X(\Sigma^0) - 6C_m(d, d) + 2C_e(d, d) - \frac{18}{30} = \frac{19}{64} - \frac{1}{10} \\
C_m(d, s) &= \frac{\frac{13}{30} - X_{eff}(\Sigma^+) + \Delta(\Sigma)}{4} + C_m(d, d) + C_e(d, d) - C_e(d, s) \\
&= -\frac{7}{16} + \frac{1}{3} + \frac{1}{3 \cdot 256} \simeq -\frac{1}{6}
\end{aligned} \tag{116}$$

(recall the definitions  $E_m = 3C_m(d, d)$  and  $C_e(d, d) = C_e$ ). The knowledge of

$\Sigma^*$  masses would allow the deduction of  $\Delta(\Sigma^*)$  and prediction of splittings inside  $\Sigma^*$  multiplet.  $C_m(d, s) \simeq -\frac{1}{6}$  has much larger absolute value than  $C_m(d, d) \simeq \frac{1}{32}$  but has opposite sign whereas  $C_e(d, s) = \frac{19}{64} - \frac{1}{10}$  is of same order of magnitude as  $C_e(d, d) \simeq \frac{1}{6}$ .

The splittings in  $\Xi^*$  and  $\Xi$  multiplets are better known and one can deduce the values of the parameters  $C_m(s, s)$ ,  $C_e(s, s)$ ,  $\Delta(\Xi)$  and  $\Delta(\Xi^*)$  but no predictions are possible. After some algebra one obtains

$$\begin{aligned}
\Delta(\Xi) &= \frac{X_{eff}(\Xi^0) + 2X_{eff}(\Xi^-) - \frac{18}{30} - C_e(d, s)}{3} = \frac{15}{64} - \frac{1}{6} + \frac{1}{3 \cdot 64} \simeq -\frac{5}{12} \\
C_m(s, s) &= \frac{X_{eff}(\Xi^{*0}) - X_{eff}(\Xi^0) - X_{eff}(\Xi^{*-}) + X_{eff}(\Xi^-) + 25C_m(d, s)}{5} \\
&\simeq -\frac{5}{32} + \frac{13}{60 \cdot 64} \\
C_e(s, s) &= 3C_m(s, s) + X_{eff}(\Xi^{*-}) - \Delta(\Xi) = \frac{1}{2} + \frac{1}{6} - \frac{19}{60 \cdot 64} + \simeq \frac{2}{3} \\
\Delta(\Xi^*) &= \Delta(\Xi) + 5C_m(d, s) + 4C_m(s, s) + X_{eff}(\Xi^{*0}) - X_{eff}(\Xi^0) \\
&\simeq -\frac{17}{64} - \frac{1}{6} + \frac{1}{20 \cdot 64} \simeq -\frac{5}{12}
\end{aligned} \tag{117}$$

Notice that  $C_m(s, s)$  does not differ numerically very much from  $C_m(d, s)$ . In case of  $\Lambda$  and  $\Omega$  one multiplet shifts  $\Delta(\lambda)$  and  $\Delta(\Omega)$  are the only unknown quantities and can be derived from the known values of  $X_{eff}$ . Rather frustratingly, no actual tests for the scenario are obtained due to the poor knowledge of  $\Sigma^*$  splittings.

#### 4.7.2 Mesonic case

The following table represents contributions to mesonic second order term in mass squared and also the values of  $X_{eff}$  deduced from mass fit.

meson	$\frac{\Delta X(magn)}{E_m}$	$\frac{\Delta X(coul)}{C_e}$	X	$\Delta X^I/\Delta_0^I$	$\Delta X^s/\Delta^s$	$64X_{eff}$
$\pi^0$	$\frac{5}{4}$	$-\frac{5}{2}$	$\frac{13}{30}$	$-\frac{1}{4}$	$-\frac{3}{4}$	24
$\pi^+$	-1	2	$\frac{13}{30}$	$-\frac{1}{4}$	$-\frac{3}{4}$	26
$\rho^0$	$-\frac{5}{12}$	$-\frac{5}{2}$	$\frac{13}{30}$	$-\frac{1}{4}$	$\frac{1}{4}$	29
$\rho^+$	$\frac{2}{6}$	2	$\frac{13}{30}$	$-\frac{1}{4}$	$\frac{1}{4}$	24
$\eta$	$\frac{5}{4}$	$-\frac{5}{2}$	$\frac{13}{30}$	$-\frac{1}{4}$	$-\frac{3}{4}$	19
$\omega$	$-\frac{5}{12}$	$-\frac{5}{2}$	$\frac{13}{30}$	$\frac{3}{4}$	$\frac{1}{4}$	51
$K^+$	-1	2	$\frac{13}{30}$	0	$-\frac{3}{4}$	6
$K^0$	$\frac{1}{2}$	-1	$-\frac{10}{30}$	0	$-\frac{3}{4}$	11
$K^{*+}$	$\frac{1}{3}$	2	$\frac{13}{30}$	0	$\frac{1}{4}$	39
$K^{*0}$	$-\frac{1}{6}$	-1	$-\frac{10}{30}$	0	$\frac{1}{4}$	49
$\eta'$	$\frac{1}{2}$	-1	$-\frac{10}{30}$	$\frac{3}{4}$	$-\frac{3}{4}$	10
$\Phi$	$-\frac{1}{6}$	-1	$-\frac{10}{30}$	$\frac{3}{4}$	$\frac{1}{4}$	44
$D^+$	-1	2	$\frac{13}{30}$	0	$-\frac{3}{4}$	63
$D^0$	$\frac{1}{2}$	-1	$-\frac{10}{30}$	0	$-\frac{3}{4}$	39
$D^{*+}$	$\frac{1}{3}$	2	$\frac{13}{30}$	0	$\frac{1}{4}$	24
$D^{*0}$	$-\frac{1}{6}$	-1	$-\frac{10}{30}$	0	$\frac{1}{4}$	9
$\eta_c$	$\frac{1}{2}$	-1	$-\frac{10}{30}$	$\frac{3}{4}$	$-\frac{3}{4}$	29
$\Psi$	$-\frac{1}{6}$	-1	$-\frac{10}{30}$	$\frac{3}{4}$	$\frac{1}{4}$	44

Table 4.21. Various second order contributions to meson masses. The notation  $E_m = 2C_m = \frac{D_m}{18}$  and  $C_e = \frac{D_e}{9}$  (slightly different as compared with the baryonic case) have been used.

It is convenient to write second order correction to meson mass as

$$\begin{aligned}
X &= \Delta X(Coul) + \Delta X(magn) + \Delta_0 + \Delta X^I + \Delta^{spin} \\
\Delta X^I &= -\frac{1}{2}\Delta^I(I(I+1) - \frac{3}{2}) \\
\Delta X^{spin} &= \frac{1}{2}\Delta^{spin}(J(J+1) - \frac{3}{2})
\end{aligned} \tag{118}$$

$\Delta^0$  is constant shift coming from sea and color Coulombic interaction.  $\Delta^I$  is a shift coming from the interaction between same generation isospins and has same dependence on isospin as color hyper fine splitting on  $\Delta^{spin}$  on spin.

The values of the parameters  $C_e, C_m, \Delta_0, \Delta^I, \Delta^{spin}$  for nonstrange quarks can be determined by comparing predictions with the actual values of  $X_{eff,n}$  and one obtains for nonstrange mesons the following equations determining the parameter values.

$$\begin{aligned}
\Delta_0 &= \frac{1}{12}(X_{eff}(K^+) + 2X_{eff}(K^0) + 3(X_{eff}(K^{*+}) + 2X_{eff}(K^{*0})) - \frac{7}{90}) \\
&= \frac{1}{2} + \frac{1}{64 \cdot 12} - \frac{7}{90} \\
C_m(d, d) &= \frac{3}{14}(X_{eff}(\pi^0) - X_{eff}(\rho^0) - X_{eff}(\pi^+) + X_{eff}(\rho^+)) = -\frac{3}{128} \\
\Delta^{spin}(d, d) &= -X_{eff}(\pi^+) + X_{eff}(\rho^+) - \frac{8C_m}{3} = \frac{1}{32} \\
C_e(d, d) &= C_m(d, d) - \frac{2(X_{eff}(\pi^0) - X_{eff}(\pi^+))}{9} = \frac{1}{9 \cdot 16} - \frac{3}{128} \\
\Delta^I(d, d) &= 4(-2C_m(d, d) + 2C_e(d, d) - X_{eff}(\pi^+) + \frac{13}{30} - \frac{3\Delta^{spin}(d, d)}{4} + \Delta_0) \\
&= -\frac{7}{32} - \frac{1}{45} + \frac{1}{3 \cdot 64}
\end{aligned} \tag{119}$$

The formulas are obtained from  $X_{eff}(K^+) + 2X_{eff}(K^0) + 3(X_{eff}(K^{*+}) + 2X_{eff}(K^{*0}))$ ,  $\pi^0 - \rho^0$  and  $\pi^+ - \rho^+$  mass differences and  $\pi^+$  mass. The absolute value of  $C_e = -19/(9 \cdot 128)$  is considerably smaller than its baryonic counterpart  $C_e(d, d, B) \simeq 1/6$ : a possible interpretation is that pion radius is about 10 times larger than proton radius.  $C_m \simeq -3/128$  is much smaller than  $C_m(d, d, B) \simeq 2/9$ . The value of  $C_e$  and  $C_m$  are negative, which looks peculiar.  $C_m$  and  $C_e$  are however determined only modulo integer (multiple of 6 for  $C_m$ ) and this multiple gives extremely small contribution to mass squared. One cannot exclude the possibility that all parameters are nearly integers and that the values deduced for them correspond to the deviations from integers.

One can predict the value of  $X(\omega)$ :

$$X(\omega) = X(\rho) - \Delta^I \simeq \frac{29}{64} - \Delta^I = \frac{43}{64} + \frac{1}{45} + \frac{1}{3 \cdot 64}:$$

The real counterpart  $(pX(\omega))_R \simeq \frac{46}{64}$  is not far from the experimental value  $\frac{51}{64}$ .  $X(\eta)$  can be predicted reliably if the effects resulting from mixing with  $\eta'$  and  $\eta_c$  are negligible. One gets  $X(\eta) = X(\pi^0) + \Delta^I \simeq \frac{10}{64} - \frac{1}{45} + \frac{1}{3 \cdot 64}$ , whose

real is certainly too too small as compared with the actual value  $\frac{19}{64}$ . The real counter part of the second order term is sensitive to mixing effects so that the result is not a catastrophe.

For strange and charmed mesons the parameters  $C_m(d, s), C_e(d, s)$  and  $\Delta^{spin}(d, s)$  can be deduced from  $K^* - K^+$ ,  $K^{*0} - K^0$  and  $K^+$  mass assuming  $\Delta^I$  to be negligibly small ( $\Delta^I$  contributes to  $K^0(CP = 1) - K^0(CP = -1)$  mass difference). Similar procedure can be used to deduce  $C_m(d, c), C_e(d, c)$  and  $\Delta^{spin}(d, c)$ . In this manner one obtains

$$\begin{aligned}
C_m(d, s) &= (X(K^{*+}) - X(K^+)) - (X(K^{*0}) - X(K^0)) = -\frac{5}{64} \\
\Delta^{spin}(d, s) &= \frac{1}{3}((X(K^{*+}) - X(K^+)) + 2(X(K^{*0}) - X(K^0))) = \frac{9}{16} + \frac{1}{192} \\
C_e(d, s) &= C_m(d, s) + \frac{3}{8}\Delta^{spin}(d, s) + \frac{X(K^+)}{2} - \frac{13}{60} + \frac{\Delta_0}{2} \\
&= \frac{12}{64} + \frac{1}{30} - \frac{1}{192}
\end{aligned} \tag{120}$$

The value of  $C_m(d, s)$  has same order of magnitude and sign as  $C_m(d, d)$  but  $C_e(d, s)$  and  $\Delta^{spin}$  are much larger than for nonstrange mesons.

For charmed mesons  $D^+, D^0, D^{*+}, D^{*0}$  similar formulas apply and give the following values for the parameters

$$\begin{aligned}
C_m(d, c) &= -\frac{9}{64} \\
\Delta^{spin}(d, c) &= \frac{7}{64} + \frac{1}{192} \\
C_e(d, c) &= \frac{25}{64} + \frac{1}{30} + \frac{1}{6 \cdot 64}
\end{aligned} \tag{121}$$

(d,c)-elements are if same sign as (d,s)- elements but larger.

The values of  $X_{eff}$  for  $\eta, \eta', \eta_c, \Phi$  and  $\Psi$  involve besides the known mixing angles the parameters  $C_m(i, i), C_e(i, i), \Delta^{spin}(i, i), \Delta^I(i, i), i = s, c, 8$

parameters altogether, so that masses can be reproduced with several choices of parameter values but predictions are not possible.

In the previous calculations the assumption  $\Delta^I(d, s) = 0, \Delta^I(d, c) = 0$  has been made. The nonvanishing of this parameter induces splitting between  $K^+, K^0, K^0(CP = -1)$   $I = 1$  multiplet (same for  $D$  mesons) and  $K^0(CP = -1)$   $I = 0$  multiplet and explains the mass splitting of neutral kaon system. The value of the splitting parameter  $\Delta^I(d, s)$  can be estimated:

$$\begin{aligned}\Delta^I(d, s) &= \frac{2m_K \Delta m}{1024^2 m_0^2} \simeq \frac{2\delta m}{m_K} s_{eff}(K) \sim 10^{-13} \sim 2^{-44} \sim \frac{1}{\sqrt{M_{89}}} \\ s_{eff}(K) &= 5\end{aligned}\tag{122}$$

Probably the appearance of  $M_{89}$  is not an accident: the decay of kaon to intermediate gauge boson pair explains mass splitting in standard model and  $M_{89}$  is the condensation level of intermediate gauge bosons. It should be noticed that the well known  $\Delta I = 1/2$  rule becomes conservation law for electroweak isospin in TGD: for instance the decays of  $K^0(CP = -1)$  to two pions are strongly suppressed since initial state corresponds to  $I_{ew} = 0$  state and final state has  $I_{ew} = 1$  by Bose statistics. The observed decays are made possible by CP breaking implying a small mixing of  $I_{ew} = 0$  and  $I_{ew} = 1$  states.

## 5 The observed top quark and $M_{89}$ physics?

Top quark form the only exception in the nice general picture. The TGD:eish predictions for the top mass for  $k = 89$  and  $k = 97$  levels and are  $m_t(89) \simeq 871 \text{ GeV}$  and  $m_t(97) \simeq 60.7 \text{ GeV}$  to be compared with the mass  $m_t(obs) \simeq 174 \text{ GeV}$  of the observed top candidate. The study of CKM matrix led to the cautious conclusion that only the experimental top candidate is consistent with CP breaking in  $K - \bar{K}$  system so that observed top does not seem to correspond to neither  $k = 97$  nor  $k = 89$  condensate level. A possible explanation of the discrepancy is mixing of primary condensate levels: observed top corresponds to the actual top, for which small mixing of condensate level  $k = 97$  with condensate level  $k = 89$  takes place:

$$t = \cos(\Phi)t_{97} + \sin(\Phi)t_{89}$$

$$\sin^2(\Phi) = \frac{m_t^2 - m_t^2(97)}{m_t^2(89) - m_t^2(97)} \sim .036 \quad (123)$$

The value of the mixing angle is rather small and means that top spends less than 4 per cent of its time on  $k = 89$  level.

### **Acknowledgements**

It would not been possible to carry out this work without the concrete help of my friends in concrete problems of the everyday life and I want to express my gratitude to them. Also I want to thank J. Arponen, R. Kinnunen and J. Maalampi for practical help and interesting discussions.

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Mass fit for hadrons

The tables below gives the parameters  $(s(H), n)$  for hadrons in the fit

$$\begin{aligned}
M^2(H) &= \left(s(H) + \frac{n(H)}{64}\right) M_0^2 \frac{M_{127}}{M_{107}} \\
M_0^2 &= \frac{m_e^2}{5 + \frac{2}{3} + \frac{1}{64}} \\
M^2(H) &\simeq \left(s + \frac{n}{64}\right) M_0^2
\end{aligned} \tag{124}$$

The approximate representation  $M^2 \simeq (s + \frac{n}{64})M_0^2$  is suggested by the small quantum number limit for Ramond representation and provides an excellent fit of baryon masses, the relative errors being below  $2^{-11}$ . For light mesons the errors are few per cent. For  $\Delta^-$  one must have  $m \leq 1237.5 \text{ MeV}$  instead of  $m(\Delta^{++} + (7.9 \pm 6.8) \text{ MeV})$  in order to obtain same value of  $s(eff)$  for the entire multiplet. The precise measurement of  $\Delta^-$  mass obviously gives a crucial test for the scenario.

baryon	$m_{exp}/MeV$	s	$n$	$10^3(\Delta M/M)$
$p$	938.2796	18	25	.13
$n$	939.5731	18	28	.82
$\Delta^{++}$	1231	31	41	.6
$\Delta^+$	1234.8	31	54	.9
$\Delta^0$	1233.6	31	50	.8
$\Delta^-$	$\leq 1237.5$	31	$\leq 63$	?
$\Lambda$	1115.60	25	63	1.112
$\Sigma^+$	1189.37	29	35	.69
$\Sigma^0$	1192.37	29	45	.88
$\Sigma^-$	1197.35	29	60	.99
$\Sigma^{*+}$	1385	40	4	.076
$\Xi^0$	1314.9	36	7	.12
$\Xi^-$	1321.29	36	29	.32
$\Xi^{*0}$	1531.8	49	0	-.029
$\Xi^{*-}$	1534.97	49	13	.11
$\Omega^-$	1672.2	58	25	.16
$\Lambda_c$	2282.2	108	50	.27
$\Lambda_b$	5425	614	41	.04

Table 5.1. Mass fit for baryons.

meson	$m_{exp}/MeV$	s	X	$10^2(\Delta M/M)$
$\pi^0$	134.9645	0	24	2.5
$\pi^+$	139.5688	0	26	3.1
$\rho^0$	772	12	29	.15
$\rho^+$	770	12	24	.07
$\omega$	783	12	51	.18
$\eta$	548.9	6	19	.2
$K^+$	493.707	5	6	.094
$K^0$	497.7	5	11	.099
$K^{*+}$	891.77	16	39	.13
$K^{*0}$	896.05	16	49	.14
$\eta'$	957.6	19	10	.04
$\Phi$	1019	21	44	.1
$D^+$	1869.4	72	63	.05
$D^0$	1864.7	72	39	.02
$D^{*+}$	2010.1	84	24	.01
$D^{*0}$	2007.2	84	9	.006
$F$	2021	85	19	.0097
$\eta_c$	2980	185	29	.006
$\Psi$	3100	200	45	.01
$B$	5270	580	1	.0003
$Y$	9460	1868	61	.002

Table 5.2. Mass fit for mesons. The relative errors for pions are order 3 per cent.